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Minimization of Sensor Activation in Decentralized Fault Diagnosis of Discrete Event Systems

Xiang Yin and Stéphane Lafortune

Abstract— **We investigate the problem of dynamic sensor activation for decentralized fault diagnosis in partially-observed discrete-event systems, where the system is monitored by a set of agents. The sensors of each agent can be turned on/off online dynamically according to a sensor activation policy. The goal is to find a language-based minimal sensor activation policy for each agent such that the agents can still diagnose, as a team, all fault occurrences. A novel approach to solve this problem is proposed. We adopt a *person-by-person* approach to decompose this decentralized minimization problem into two centralized constrained minimization problems. Each centralized constrained minimization problem is then reduced to a fully centralized sensor activation problem that is solved effectively by an algorithm presented in a related contribution. The solution obtained is provably person-by-person minimal with respect to the system language.**

I. INTRODUCTION

Fault diagnosis is an important task in complex automated systems. In this paper, we investigate the problem of decentralized fault diagnosis in Discrete Event Systems (DES) that operate under dynamic observations. In this context, the system is monitored by a set of agents that act as a team to diagnose all fault occurrences. Each agent makes observations online through its sensors; these sensors can be turned on/off online dynamically during the evolution of the system according to a sensor activation policy that depends on the agent's observations. Due to energy, bandwidth, or security constraints, sensors activations are "costly". Therefore, in order to reduce sensor-related costs, it is of interest to minimize, in some technical sense, the sensor activations of each agent while maintaining the desired property of decentralized diagnosability.

The problem of sensor optimization in DES was initially studied in [1], [2], [3]; the goal in these works was to find an optimal set of observable events that is fixed for the entire execution of the system and enforces a given DES-theoretic property. This problem is referred to as optimal sensor selection for *static observations*. In the context of *dynamic observations*, where sensors can be turned on/off dynamically, the corresponding problem of optimal sensor activation has also received a lot of attention in the literature; see, e.g., [4], [5], [6], [7], [8], [9], [10], [11]. For example, in [4], [5] the problem of *centralized* dynamic sensor activation for enforcement of different diagnosability properties was solved optimally w.r.t. numerical cost criteria. In [6], dynamic sensor activation for enforcement of centralized diagnosability was also studied, but in the context of a logical optimality criterion.

In many large scale systems, the information structure is decentralized due to the distributed nature of the sensors. In the decentralized diagnosis problem considered in [12], the system is monitored by a set of local agents that work as a team in order to diagnose every occurrence of fault events. In [6], the problem of dynamic sensor activation for decentralized diagnosis is studied. Specifically, a "window-based partition" approach is proposed in order to obtain a solution. However, a drawback of this approach is that the solution obtained is only optimal w.r.t a finite (restricted) solution space and may not be language-based optimal in general. In other words, by enlarging the solution space by refining the state space of the system model, better solutions could be obtained in principle. In [7], the problem of dynamic sensor activation for decentralized control is also studied, where the solution obtained is again optimal w.r.t. a finite solution space. To the best of our knowledge, the problem of *language-based* sensor optimization for *decentralized* diagnosis has remained an open problem, as is mentioned in the recent survey [13].

In this paper, we propose a new approach to tackle the problem of dynamic sensor activation for the purpose of decentralized diagnosis. Specifically, we adopt a *person-by-person* approach (see, e.g., [14] and the references therein) to decompose the decentralized minimization problem to two consecutive centralized minimization problems. We consider the case of two agents, Agents 1 and 2. We first minimize the sensor activation policy for Agent 1 by keeping the policy of Agent 2 fixed. Then, we fix Agent 1's sensor activation policy to the one obtained and solve the same minimization problem but for Agent 2. Essentially, we solve two centralized *constrained* minimization problems, since we need to take the other agent's information into account when we minimize the decisions of an agent. Each centralized constrained minimization problem is then reduced to a problem that is solved effectively by a new algorithm presented in recent related work [15].

In general, a person-by-person approach in team decision problems may not terminate in a finite number of steps, since we may need to iterate between the two constrained minimization problems. However, we show that for the problem under consideration in this paper, such iterations are not required due to a certain type of *monotonicity* that arises. Moreover, we prove that the solution obtained by our

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Xiang Yin and Stéphane Lafortune are with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA. {xiangyin,stephane}@umich.edu.

procedure is minimal w.r.t. the system language (i.e., over an infinite set in general), in contrast to the works reviewed above where minimality was with respect to a finite solution space. In the DES literature, the person-by-person approach has also been applied to the decentralized control problem [16] and to the decentralized communication problem [17], [18], [19]. However, to the best of our knowledge, it has not been applied so far to decentralized sensor activation.

Due to space constraints, all proofs have been omitted.

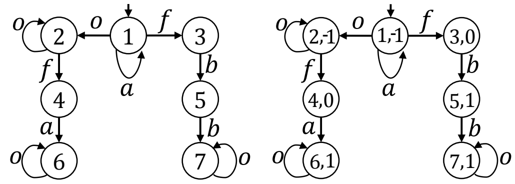
II. Preliminaries

1) System Model: We consider a DES modeled as a deterministic finite-state automaton (DFA) $G=(Q,\Sigma,\delta,q_0)$, where Q is the finite set of states, Σ is the finite set of events, $\delta: Q\times\Sigma\rightarrow Q$ is the partial transition function and q_0 is the initial state. The function δ is extended to $Q\times\Sigma^*$ in the usual way (see, e.g., [20]). For any $q\in Q, s\in\Sigma^*$, we write $\delta(q,s)$ as $\delta(s)$ if $q=q_0$. The prefix-closure of language $L\subseteq\Sigma^*$ is $\overline{L}=\{s\in\Sigma^*:\exists w\in\Sigma^* \text{ s.t. } sw\in L\}$. We say that L is prefix-closed if $L=\overline{L}$. The behavior of the system G is described by the prefix-closed language $\mathcal{L}(G)=\{s\in\Sigma^*:\delta(q_0,s)!\}$, where ! means is defined. We say that language L is live if $\forall s\in L,\exists\sigma\in\Sigma: s\sigma\in L$. Hereafter, we assume that $\mathcal{L}(G)$ is live, which is a standard assumption in diagnosability analysis.

2) Information Mapping: We consider a general dynamic observations setting, where the observability properties of events can be controlled by a *sensor activation policy* during the evolution of the system. Let $\Sigma_o\subseteq\Sigma$ be the set of events that can become observable by activating some sensors. A sensor activation policy is defined as a deterministic labeled automaton $\Omega=(A,L)$, where $A=(Q_A,\Sigma_o,\delta_A,q_{0,A})$ is a deterministic automaton and $L: Q_A\rightarrow 2^{\Sigma_o}$ is a labeling function that specifies the current set of "observable" events within Σ_o. Specifically, for any $s\in\Sigma_o^*$, $L(\delta_A(s))$ denotes the set of events that are *monitored* after observing s. While an event is monitored, any occurrence of it will be observed by the diagnoser. In other words, after string s, events in $\Sigma_o\setminus L(\delta_A(s))$ are currently "unobervable" (i.e., their sensors are turned off). Moreover, the pair (A,L) needs to satisfy the constraint that $\forall q\in Q_A,\forall\sigma\in\Sigma_o:\sigma\in L(q)\Leftrightarrow\delta_A(q,\sigma)!$. This condition says that the sensor activation policy can only be updated (by updating the state of A) when a monitored event occurs. In general, Q_A could be an infinite set. However, we will show later that the optimal sensor activation policies of interest in this paper can always be constructed with finite state spaces.

We say that the observations are *static* if the set of observable events is fixed a priori. We denote by Ω_{Σ_o} the corresponding sensor activation policy for the static observation with the set of observable events Σ_o. Specifically, $\Omega_{\Sigma_o}=(A,L)$ is given by: 1) $Q_A=\{q_{0,A}\}$; 2) $\forall\sigma\in\Sigma_o:\delta_A(q_{0,A},\sigma)=q_{0,A}$; and 3) $L(q_{0,A})=\Sigma_o$. Given a sensor activation policy $\Omega=(A,L)$, we define the corresponding information mapping $P_\Omega:\mathcal{L}(G)\rightarrow\Sigma_o^*$ recursively as follows:

$$P_\Omega(\epsilon)=\epsilon,\quad P_\Omega(s\sigma)=\begin{cases}P_\Omega(s)\sigma & \text{if } \sigma\in L(\delta_A(P_\Omega(s)))\\ P_\Omega(s) & \text{if } \sigma\notin L(\delta_A(P_\Omega(s)))\end{cases}$$



(a) Original system G (b) Augmented system $\tilde{G}$

Fig. 1. System Model.

For any language $L\subseteq\Sigma^*$, we define $P_\Omega(L)=\{t\in\Sigma_o^*:\exists s\in L \text{ s.t. } P_\Omega(s)=t\}$. For any two sensor activation policies $\Omega=(A,L)$ and $\Omega'=(A',L')$, we write that $\Omega'\subseteq\Omega$ if

$$\forall s\in\mathcal{L}(G): L'(\delta_{A'}(P_{\Omega'}(s)))\subseteq L(\delta_A(P_\Omega(s)))\qquad(1)$$

and write that $\Omega'\subset\Omega$ if

$$[\Omega'\subseteq\Omega]\wedge[\exists s\in\mathcal{L}(G):L'(\delta_{A'}(P_{\Omega'}(s)))\subset L(\delta_A(P_\Omega(s)))]\ (2)$$

3) The Observer: Let $G=(Q,\Sigma,\delta,q_0)$ be the system automaton and $\Omega=(A,L), A=(Q_A,\Sigma_o,\delta_A,q_{0,A})$ be a sensor activation policy. The observer for G under Ω is $Obs_\Omega(G)=(X,\Sigma_o,f,x_0)$, where $X\subseteq 2^Q\times Q_A$ is the state space and for any state $x\in X$, we write $x=(I(x),A(x))$ where $I(x)\in 2^Q$ and $A(x)\in Q_A$. The partial transition function of the observer is denoted by $f: X\times\Sigma_o\rightarrow X$ and is defined as follows. For any $x=(i,q), x'=(i',q')\in X$ and $\sigma\in\Sigma_o$, $f(x,\sigma)=x'$ iff

$$\begin{cases} q'=\delta_A(q,\sigma)\\ i'=UR_{L(q')}(Next_\sigma(i))\end{cases}\qquad(3)$$

where for any $i\in 2^Q,\sigma\in\Sigma_o$ and $\theta\in 2^{\Sigma_o}$,

$$Next_\sigma(i)=\{q_1\in Q:\exists q_2\in i \text{ s.t. } \delta(q_2,\sigma)=q_1\}$$
$$UR_\theta(i)=\{q_1\in Q:\exists q_2\in i,\exists s\in(\Sigma\setminus\theta)^* \text{ s.t. } \delta(q_2,s)=q_1\}$$

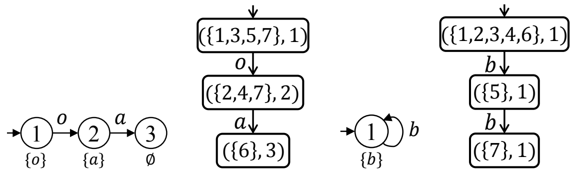
Intuitively, $Next_\sigma(i)$ is the set of states that can be reached from some state in i immediately after observing σ and $UR_\theta(i)$ is the set of states that can be reached unobservably from some state in i under the set of monitored events θ. Finally, the initial state of $Obs_\Omega(G)$ is $x_0=(UR_{L(q_{0,A})}(\{q_0\}),q_{0,A})$. For simplicity, we only consider the reachable part of $Obs_\Omega(G)$. By the above definition, we have that $\mathcal{L}(Obs_\Omega(G))=P_\Omega(\mathcal{L}(G))$.

We define the *state estimator function* (or simply "state estimator") under ω, $\mathcal{E}_\omega^G:\mathcal{L}(G)\rightarrow 2^Q$, as follows upon the occurrence of $s\in\mathcal{L}(G)$:

$$\mathcal{E}_\Omega^G(s):=\{q\in Q:\exists t\in\mathcal{L}(G) \text{ s.t. } P_\Omega(s)=P_\Omega(t)\wedge\delta(q_0,t)=q\}$$

By a simple induction (see, e.g., [10]), we can show that, for any $s\in\mathcal{L}(G)$, $I(f(P_\Omega(s)))=\mathcal{E}_\Omega^G(s)$, i.e., the state components of the observer state reached upon $P_\Omega(s)$ is the state estimator value after s.

Example 2.1: Consider the system G in Fig. 1(a). Let $\Sigma_{o,1}=\{o,a\}$ and $\Sigma_{o,2}=\{b\}$ be two sets of observable events. As shown in Fig. 2(a), Ω_1 is a sensor activation policy with the set of observable events $\Sigma_{o,1}$. The labeling function is specified by the set of events associated with each state in the figure. Initially, event o is monitored by Ω_1. Once o is observed, Ω_1 changes to monitor event a. Finally, Ω_1



(a) Ω_1 (b) $Obs_{\Omega_1}(G)$ (c) Ω_2 (d) $Obs_{\Omega_2}(G)$

Fig. 2. Examples of sensor activation policies and observers

turns all sensors off when a is observed. The corresponding observer $Obs_{\Omega_1}(G)$ is shown in Fig. 2(b). For example, for the string $oofa \in \mathcal{L}(G)$, we have that $P_{\Omega_1}(oofa) = oa$ and $I(f(oa)) = \{6\} = \mathcal{E}_{\Omega}^{G}(oofa)$. Similarly, Fig. 2(c) shows a sensor activation policy Ω_2 with the set of observable events $\Sigma_{o,2}$. Clearly, Ω_2 always monitors all events in $\Sigma_{o,2}$, i.e., $\Omega_2 = \Omega_{\Sigma_{o,2}}$. Therefore, the observer $Obs_{\Omega_2}(G)$ shown in Fig. 2(d) is the standard observer (see, e.g, [20]) if we ignore the second component of each state. ■

III. Decentralized Fault Diagnosis

A. Problem Formulation

In the decentralized fault diagnosis problem, the system is monitored by a set of agents that work as a team in order to diagnose all occurrence of fault events. We denote by $\mathcal{I}$ the index set of the agents. In this paper, we consider the case where two agents are involved, i.e., $\mathcal{I} = \{1, 2\}$. For each agent $i \in \{1, 2\}$, we denote by Ω_i its sensor activation policy and by $\Sigma_{o,i}$ its set of observable events in Ω_i. We define the pair of sensor activation policies as $\bar{\Omega} = [\Omega_1, \Omega_2]$. As defined in [7], [6], the inclusion $\bar{\Omega}' \subseteq \bar{\Omega}$ means that $\forall i \in \{1, 2\} : \Omega_i' \subseteq \Omega_i$ and the strict inclusion $\bar{\Omega}' \subset \bar{\Omega}$ means that $[\bar{\Omega}' \subseteq \bar{\Omega}] \wedge [\exists i \in \{1, 2\} : \Omega_i' \subset \Omega_i]$.

For the sake of a simpler presentation, we assume hereafter that there is a single fault event (or class), as in the prior literature on sensor activation for enforcement of diagnosability. The optimization methodology developed in this paper depends on testing the property of K-diagnosability; in the case of multiple fault events, K-diagnosability holds if it holds for each fault event (or class) individually. Let $e_d \in \Sigma \setminus (\cup_{i=1,2}\Sigma_{o,i})$ be the fault event that we want to diagnose. We denote by $\Psi(e_d) = \{se_d \in \mathcal{L}(G) : s \in \Sigma^*\}$ the set of strings that end with e_d. We write that $e_d \in s$ if $\overline{\{s\}} \cap \Psi(e_d) \neq \emptyset$. We recall the definition of K-codiagnosability under dynamic observations from [8], [9], which requires that all occurrences of e_d be detected unambiguously within K-steps after each occurrence.

Definition 1: (K-Codiagnosability). Let $K \in \mathbb{N}$. We say that live language $\mathcal{L}(G)$ is K-codiagnosable w.r.t. $\bar{\Omega}$ and e_d if

$$(\forall s \in \Psi(e_d))(\forall t \in \mathcal{L}(G)/s)[|t| \geq K \Rightarrow \mathcal{CD}]$$

where the codiagnosability condition $\mathcal{CD}$ is

$$(\exists i \in \{1, 2\})(\forall w \in \mathcal{L}(G))[P_{\Omega_i}(w) = P_{\Omega_i}(st) \Rightarrow e_d \in w].$$

We are now ready to formulate the problem of minimal sensor activation for decentralized diagnosis.

Problem 1: Let G be the system with fault event e_d. For each agent $i \in \{1, 2\}$, let $\Sigma_{o,i} \subseteq \Sigma$ be the set of observable events. Find sensor activation policies $\bar{\Omega}^* = [\Omega_1^*, \Omega_2^*]$ s.t.:

C1. $\mathcal{L}(G)$ is K-codiagnosable w.r.t. $\bar{\Omega}^*$ and e_d.

C2. $\bar{\Omega}^*$ is minimal, i.e., there does not exist another $\bar{\Omega}' \subset \bar{\Omega}^*$ that satisfies (C1).

To guarantee that Problem 1 has a solution, we assume that $\mathcal{L}(G)$ is K-codiagnosable w.r.t. $[\Omega_{\Sigma_{o,1}}, \Omega_{\Sigma_{o,2}}]$ and e_d, i.e., the fault can be diagnosed when all sensors are continuously on.

Remark 3.1: In [6], a "sub-optimal" solution to Problem 1 is provided, in the sense that the solution found therein is minimal among all solutions over a *given* finite partition of the language $\mathcal{L}(G)$. In principle, the solution found in [6] could be improved by employing a finer partition of $\mathcal{L}(G)$ and repeating the optimization procedure. In this paper, we are aiming for a *language-based* minimal solution, in the sense that the notion of strict inclusion of sensor activation policies is defined in terms of the strings in $\mathcal{L}(G)$. In other words, we do not impose, a priori, any constraints on the state space of each Ω_i. Hence, no better solution can be obtained by refining the state space of G and repeating the solution procedure. To the best of our knowledge, such a language-based optimal solution to Problem 1 has never been reported in the literature. We will see that there always exists an $\bar{\Omega}^*$ solving Problem 1 where each Ω_i^* has a finite realization.

B. Augmented Automaton

To simplify the ensuing analysis, we provide a state-based characterization of K-codiagnosability. First, we define the K-augmented automaton $\tilde{G} = (\tilde{Q}, \Sigma, \tilde{\delta}, \tilde{q}_0)$, where $\tilde{Q} = Q \times \{-1, 0, 1 \ldots, K\}$, $\tilde{q}_0 = (q_0, -1)$ and the partial transition function $\tilde{\delta} : \tilde{Q} \times \Sigma \to \tilde{Q}$ is defined by: for any $(q, n) \in \tilde{Q}$ and $\sigma \in \Sigma$, we have

$$\tilde{\delta}((q,n),\sigma) = \begin{cases} (\delta(q,\sigma), -1), & \text{if } n = -1 \text{ and } \sigma \neq e_d \\ (\delta(q,\sigma), n+1), & \text{if } \begin{array}{c}[0 \leq n < K] \text{ or} \\ [n = -1 \wedge \sigma = e_d]\end{array} \\ (\delta(q,\sigma), K), & \text{if } n = K \end{cases} \tag{4}$$

Intuitively, $\tilde{G}$ simply unfolds G by "counting" the number of steps since the fault has occurred. Since $\mathcal{L}(\tilde{G}) = \mathcal{L}(G)$, hereafter, we will analyze diagosability based on $\tilde{G}$ rather than on G. For any $q \in \tilde{Q}$, $[q]_Q$ and $[q]_n$ denote its state component and its integer component, respectively. For example, if we take $K = 1$, then the K-augmented $\tilde{G}$ for G in Fig. 1(a) is shown in Fig. 1(b).

We define an *information state* to be a subset of states and denote by $I := 2^{\tilde{Q}}$ the set of information states. We define the diagnosability function for information state $D_I : I \to \{N, C_1, C_2, F\}$ as follows. For any $x \in I$, the value $D_I(x)$ is given by:

$$D_I(x) = \begin{cases} N, & \text{if } \forall q \in x : [q]_n = -1 \\ F, & \text{if } \forall q \in x : [q]_n \geq 0 \\ C_1, & \text{if } \exists q, q' \in x : [q]_n = -1 \wedge 0 \leq [q']_n < K \\ C_2, & \text{if } \exists q, q' \in x : [q]_n = -1 \wedge \ [q']_n = K \end{cases} \tag{5}$$

Then condition $\mathcal{CD}$ in Definition 1 can be reformulated in terms of information state by

$$\mathcal{CD} \Leftrightarrow (\exists i \in \{1, 2\})[D_I(\mathcal{E}_{\Omega_i}^{\tilde{G}}(st)) = F] \tag{6}$$

C. Solution Overview

Before we formally tackle Problem 1, let us first provide a brief overview of our solution approach. We adopt the person-by-person approach that has been widely used in decentralized optimization problems. Specifically, we decompose the decentralized minimization problem to a set of centralized constrained minimization problems and for each such problem, we only attempt to minimize one agent's sensor activation policy while the other one is fixed. However, the following questions arise. First, by taking the person-by-person approach, iterations involving minimization for each agent may be required in general, and such iterations may not terminate in a finite number of steps. We will show that in our particular problem such iterations are not required. This is due of the so-called *monotonicity property* defined in [7], [6] that arises in dynamic sensor activation problems. The second question of interest is how to minimize the sensor activation policy of one agent when the policy of the other agent is fixed. This problem is different from the fully centralized minimization problem, since we should not only consider the information of the agent whose sensor activation policy we are minimizing, but we must also take into account the information available to the other agent, whose sensor activation policy is fixed. Therefore, the true information state for this minimization problem is of the form $(i, \imath), i \in 2^Q, \imath \in 2^{2^Q}$, where i is the knowledge of the agent whose sensor activation policy is being minimized and $\imath$ is *this agent's inference of the other agent's potential knowledge of the system based on that agent's own information*. To resolve this information dependency, we employ the notion of *generalized state-partition automaton*, by which we encode the second agent's knowledge into the system model. This is discussed next.

IV. Generalized State-Partition Automaton

Definition 2: (State-Partition Automaton). Let G be an automaton, Ω a sensor activation policy and $Obs_\Omega(G) = (X, \Sigma_o, f, x_0)$ the corresponding observer. We say that G is a *state-partition automaton* (SPA) w.r.t. Ω, if

$$\forall x, y \in X : I(x) = I(y) \text{ or } I(x) \cap I(y) = \emptyset \tag{7}$$

The notion of SPA was studied in [21], [22] for static observations. Clearly, if the observation map is static, i.e., there is only one state in the labeled automaton of the sensor activation policy, then the above definition reduces to the standard notion of SPA, where it is required that the states of the observer automaton are pairwise disjoint. Similarly, in our general definition, it is required that for any two observer states, their information state components should be disjoint.

Suppose that G is an SPA w.r.t. Ω and $Obs_\Omega(G) = (X, \Sigma_o, f, x_0)$ is the observer. Then for any state $q \in Q$, there exists a unique information state $\mathcal{F}(q) \in 2^Q$ such that

1) $q \in \mathcal{F}(q)$; and
2) $\exists q_A \in Q_A : (\mathcal{F}(q), q_A) \in X$.

We call this information state $\mathcal{F}(q)$ the *inference* of state q. This defines the inference function $\mathcal{F} : Q \to 2^Q$ w.r.t. Ω such that $\forall s \in \mathcal{L}(G) : [\delta(q_0, s) = q] \Rightarrow [\mathcal{F}(q) = I(f(P_\Omega(s)))]$.

Example 4.1: Consider again the system G in Fig. 1(a) and the sensor activation policy Ω_2 in Fig. 2(d). By looking at the states in $Obs_{\Omega_2}(G)$, we know that G is an SPA w.r.t. Ω_2. For example, for state 4, we know that $\mathcal{F}(4) = \{1, 2, 3, 4, 6\}$, i.e., $(\forall s \in \mathcal{L}(G))[\delta(s) = 4 \Rightarrow \mathcal{E}^G_{\Omega_1}(s) = \{1, 2, 3, 4, 6\}]$. However, G is not an SPA w.r.t. Ω_1 in Fig. 2(b), since state 7 exists in the state components of two different observer states in $Obs_{\Omega_1}(G)$. For example, we have $\delta(fbb) = \delta(fbbo) = 7$, $\mathcal{E}^G_{\Omega_1}(fbb) = \{1, 3, 5, 7\}$ but $\mathcal{E}^G_{\Omega_1}(fbb) = \{2, 4, 7\}$. ■

The inference function $\mathcal{F}$ is well defined only when G is an SPA. However, we show that for any automaton G and sensor activation policy Ω, we can always refine the state space of G such that the refined automaton is an SPA w.r.t. Ω. To see this, we first define the notion of *extended observer*. Let $Obs_\Omega(G) = (X, \Sigma_o, f, x_0)$ be the observer. Then the extended observer of G w.r.t. Ω is $Obs^+_\Omega(G) = (X, \Sigma_o, f^+, x_0)$, where $f^+ : X \times \Sigma_o \to X$ is a total function such that

$$f^+(x, \sigma) = \begin{cases} f(x, \sigma) & \text{if } f(x, \sigma) \text{ is defined} \\ x & \text{if } f(x, \sigma) \text{ is not defined} \end{cases} \tag{8}$$

The extended observer simply adds unobservable self-loops at each state in the observer such that its transition function is total. Clearly, we have that $\mathcal{L}(Obs^+_\Omega(G)) = \Sigma_o^*$. We use the notation $A \| B$ to denote the usual parallel composition operation of automata A and B.

We are now ready to show how to refine the system model such that the state-partition property holds.

Proposition 4.1: Let $G = (Q, \Sigma, \delta, q_0)$ be the system automaton, $\Omega = (A, L), A = (Q_A, \Sigma_o, \delta_A, q_{0,A})$ a sensor activation policy and $Obs^+_\Omega(G) = (X, \Sigma_o, f^+, x_0)$ the corresponding extended observer. Then $Obs^+_\Omega(G) \| G$ is an SPA w.r.t. Ω such that $\mathcal{L}(Obs^+_\Omega(G) \| G) = \mathcal{L}(G)$.

Based on the above result, since we can always construct an SPA that is language equivalent to the original automaton, in the remainder of the paper, we will assume, without loss of generality, that an automaton is an SPA when such a property is needed. For instance, in Example 4.1, if we build $Obs^+_{\Omega_1}(G) \| G$, then state 7 is split into two states and the refined system is an SPA.

V. Constrained Minimization Problem

In this section, we tackle problem of minimizing the sensor activation policy for one agent when the sensor activation policy of the other one is fixed. This problem is also referred to as the *centralized constrained minimization problem* herafter. Throughout this section, $i \in \{1, 2\}$ denotes the agent whose sensor activation policy is being minimized while $j \in \{1, 2\}, j \neq i$ denotes the other agent whose sensor activation policy is fixed.

A. Constrained Minimization Problem

Problem 2: (Centralized Constrained Minimization Problem). Let $i, j \in \{1, 2\}, i \neq j$ be two agents. Suppose that the sensor activation policy Ω_j for Agent j is fixed. Find a sensor activation policy Ω_i for Agent i such that:

C1. $\mathcal{L}(\tilde{G})$ is K-codiagnosable w.r.t. $[\Omega_1, \Omega_2]$ and

C2. For any Ω_i' satisfying (C1), we have $\Omega_i' \not\subset \Omega_i$.

The above problem is different from both the centralized and decentralized minimization problems. In the centralized minimization problem, where only one agent is involved, to maintain K-diagnosability, we need to require that

$$\forall s \in \mathcal{L}(\tilde{G}) : D_I(\mathcal{E}_{\Omega}^{\tilde{G}}(s)) \neq C_2 \tag{9}$$

In other words, the agent should always be able to distinguish states labeled by -1 and states labeled by K. However, in the decentralized diagnosis problem, it is possible that there exists a string $s \in \mathcal{L}(\tilde{G}), e_d \in s$ such that $D_I(\mathcal{E}_{\Omega_i}^{\tilde{G}}(s)) = C_2$, but $D_I(\mathcal{E}_{\Omega_j}^{\tilde{G}}(s)) = F$. In other words, to solve the constrained minimization problem for one agent, we must take the other agent's sensor activation policy into account. This issue is handled by the SPA and its inference function, as described next.

B. Formulation of Information-State-Based Property

First, we recall a general class of fully centralized sensor activation problems that is studied in [15].

Problem 3: (Centralized Sensor Minimization Problem for IS-Based Property). Let $G = (Q, \Sigma, \delta, q_0)$ be the system and $\phi : 2^Q \to \{0, 1\}$ be a function on information states. Find a sensor activation policy Ω such that

C1. $\forall s \in \mathcal{L}(G) : \phi(\mathcal{E}_{\Omega}^{G}(s)) = 1$; and

C2. For any Ω' satisfying (C1), we have $\Omega' \not\subset \Omega$.

Problem 3 is a fully centralized sensor activation problem, since only one agent is involved. This problem is studied in more detail in [15], where an algorithm is provided that solves this problem effectively by returning a finite sensor activation policy satisfying the requirements. Therefore, if we can reduce Problem 2 to Problem 3, then it means that Problem 2 can also be solved effectively and the solution will be finitely realizable. We now show that such a reduction is possible by using the notion of SPA.

Suppose that $\tilde{G} = (\tilde{Q}, \Sigma, \tilde{\delta}, \tilde{q}_0)$ is an SPA w.r.t. Ω_j and $\mathcal{F}_j : \tilde{Q} \to 2^{\tilde{Q}}$ is the corresponding inference function w.r.t. Ω_j. We define the codiagnosability function $CD_i : 2^{\tilde{Q}} \to \{0, 1\}$ for Agent i as follows. For any $x \in 2^{\tilde{Q}}$, the value $CD_i(x)$ is assigned as follows:

$$CD_i(x) = \begin{cases} 0, & \text{if } \begin{array}{c} D_I(x) = C_2 \text{ and} \\ (\exists q \in x)\, [[q]_n = K \wedge D_I(\mathcal{F}_j(q)) \neq F] \end{array} \\ 1, & \text{otherwise} \end{cases} \tag{10}$$

Let $s \in \mathcal{L}(\tilde{G}), e_d \in s$ be a faulty string and let $q = \mathcal{E}_{\Omega_i}^{\tilde{G}}(s)$ be Agent i's state estimator value for s. Since $\tilde{G}$ is an SPA w.r.t. Ω_j, we know that $\mathcal{F}_j(q) = I(f(P_{\Omega_j}(s))) = \mathcal{E}_{\Omega_j}^{\tilde{G}}(s)$. In other words, $\mathcal{F}_j(q)$ is Agent j's state estimator value for s. Therefore, if $CD_i(\mathcal{E}_{\Omega_i}^{\tilde{G}}(s)) = 0$, then it implies that $D_I(\mathcal{E}_{\Omega_k}^{\tilde{G}}(s)) = C_2, \forall k \in \{1, 2\}$, which violates the K-codiagnosability property. This observation leads to the following theorem.

Theorem 1: Suppose that $\tilde{G}$ is an SPA w.r.t. Ω_j. Then, $\mathcal{L}(\tilde{G})$ is K-codiagnosable w.r.t. $[\Omega_1, \Omega_2]$ and e_d iff

$$\forall s \in \mathcal{L}(\tilde{G}) : CD_i(\mathcal{E}_{\Omega_i}^{\tilde{G}}(s)) = 1 \tag{11}$$

Algorithm 1: D-MIN-ACT

input : $\tilde{G}, \Sigma_{o,1}$ and $\Sigma_{o,2}$
output: $\bar{\Omega}^*$

$\Omega_1^* \leftarrow \Omega_{\Sigma_{o,1}}$ and $\Omega_2^* \leftarrow \Omega_{\Sigma_{o,2}}$
for $i \in \{1, 2\}$ **do**
  $j \in \{1, 2\} \setminus \{i\}$
  if *$\tilde{G}$ is not an SPA w.r.t. Ω_j* **then**
    Reconstruct $\tilde{G}$ such that it is an SPA w.r.t. Ω_j according to Propostion 4.1.
  Fix Ω_j^*. Obtain minimal Ω_i' by solving Problem 2.
  $\Omega_i^* \leftarrow \Omega_i'$.
$\bar{\Omega}^* \leftarrow [\Omega_1^*, \Omega_2^*]$

In the above development, we assumed that $\tilde{G}$ is an SPA w.r.t. Ω_j. (Recall that this assumption always holds subject to a pre-processing procedure.) The essence of this assumption (or of the pre-processing procedure) is that we can encode Agent j's information, i.e., Ω_j, into the system model in order to reduce the constrained minimization problem for Agent i to a fully centralized minimization problem. Finally, using Theorem 1, we have the following result.

Theorem 2: Problem 2 can be effectively solved.

Since the main purpose of this paper is to show how to solve the decentralized minimization problem, the reader is referred to [15] for more details about the solution approach to Problem 3.

Example 5.1: We return to system $\tilde{G}$ in Fig. 1(b), i.e., the K-augmented system for G in Fig. 1(a) by taking $K = 1$. For the sake of brevity, we write state (q, n) in $\tilde{G}$ in the form of q^n. We want to solve the centralized minimization problem for Agent 1 subject to the constraint that Agent 2's sensor activation policy is fixed as Ω_2 in Fig. 2(c). By building the observer, we know that $\tilde{G}$ is an SPA w.r.t. Ω_2 and the inference function $\mathcal{F}_2 : \tilde{Q} \to 2^{\tilde{Q}}$ is defined by

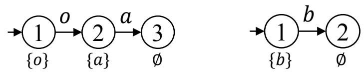
$$\mathcal{F}_2(q) = \begin{cases} \{1^{-1}, 2^{-1}, 3^0, 4^0, 6^1\}, & \text{if } q \in \{1^{-1}, 2^{-1}, 3^0, 4^0, 6^1\} \\ \{5^1\}, & \text{if } q \in \{5^1\} \\ \{7^1\}, & \text{if } q \in \{7^1\} \end{cases}$$

Let $CD_1 : 2^{\tilde{Q}} \to \{0, 1\}$ be the codiagnosability function for Agent 1 defined by Equation (11). By applying the algorithm in [15], we obtain the sensor activation policy Ω_1 in Fig. 2(a).

VI. Synthesis Algorithm

We first present an algorithm that solves the decentralized sensor activation problem by using the results we developed so far. Then we prove the correctness of the algorithm.

Our synthesis algorithm is formally presented in Algorithm D-MIN-ACT. Essentially, Algorithm D-MIN-ACT solves two centralized constrained minimization problems. First, we set Agent 2's sensor activation policy to be Ω_{Σ_2}, i.e., the most conservative one, and solve the constrained minimization problem for Agent 1. Then we fix the obtained sensor activation policy for Agent 1 and solve the constrained minimization problem for Agent 2. However, the following question arises: *After the above procedure, do we need to*



(a) Ω_1^* (b) Ω_2^*

Fig. 3. Decentralized minimal solutions

fix Agent 2's new sensor activation policy and go back to minimize Agent 1's sensor activation policy again? In other words, we need to answer whether or not iterations between two centralized constrained minimization problems are required in order to obtain a decentralized minimal solution. Hereafter, we show that such iterations are not necessary for our problem and Algorithm D-MIN-ACT indeed yields a decentralized minimal solution in the above two steps. This is because of the monotonicity property in dynamic sensor activation problem from [6].

Theorem 3: (Monotonicity Property [6]). Let G be the system, $e_d \in \Sigma$ be the fault event and $\bar{\Omega}$ and $\bar{\Omega}'$ be two sensor activation policies such that $\bar{\Omega}' \subseteq \bar{\Omega}$. Then $\mathcal{L}(G)$ K-codiagnosable w.r.t. $\bar{\Omega}'$ and e_d implies that $\mathcal{L}(G)$ is K-codiagnosable w.r.t. $\bar{\Omega}$ and e_d .

The following theorem reveals the correctness of Algorithm D-MIN-ACT.

Theorem 4: Let $\bar{\Omega}^*$ be the output of Algorithm D-MIN-ACT. Then $\bar{\Omega}^*$ solves Problem 1.

We illustrate Algorithm D-MIN-ACT by an example.

Example 6.1: Again, consider the system G in Fig. 1(a). Let $\Sigma_{o,1} = \{o, a\}$ and $\Sigma_{o,2} = \{b\}$, respectively, be the set of observable events for Agent 1 and Agent 2. Take $K = 1$. Then the K-augmented automaton is $\tilde{G}$ in Fig. 1(b). Initially, we set $\Omega_2 = \Omega_{\Sigma_{o,2}}$ and solve the constrained minimization problem for Agent 1. This is what we have shown in Example 5.1 where we obtained Ω_1^* shown in Fig. 3(a). Next, we fix Ω_1^* for Agent 1 and solve the constrained minimization problem for Agent 2. Then we obtain the sensor activation policy Ω_2^* as shown in Fig. 3(b). Clearly, we see that Ω_2^* turns all sensors off after b is observed, since once b occurs, Agent 2 will know for sure that the fault has occurred and there is no need to monitor any event. Therefore, $[\Omega_1^*, \Omega_2^*]$ is a minimal pair of sensor activation policies that ensure 1-codiagnosability. ∎

VII. Conclusion

We presented a novel approach for solving the problem of decentralized sensor activation for the purpose of fault diagnosis. We adopted a person-by-person approach to decompose the decentralized minimization problem to two consecutive centralized constrained minimization problems. The notion of generalized state-partition automaton for dynamic observations was proposed. With this notion, each centralized constrained minimization problem was reduced to a fully centralized sensor activation that is solved effectively by a new algorithm presented in recent related work. Finally, we showed that the decentralized solution obtained by our methodology is language-based minimal.

It was shown in [8], [23], [11] that K-codiagnosability and coobservability, the key property in the decentralized control problem, can be mapped to one another. Therefore, even though we focused on the problem of decentralized sensor activation for the purpose of *diagnosis* in this paper, the approach that we proposed is also applicable to the problem of decentralized sensor activation for the purpose of *control*.

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