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Improved Approaches for Verifying I-Detectability of Discrete-Event Systems

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***Abstract*— In this paper, we investigate the initial-state detection problem in the context of partially-observed discrete-event systems. Specifically, we study the verification of two properties called weak I-detectability and strong I-detectability. Weak detectability captures whether or not the initial-state of the system can be detected via some path, while strong detectability captures whether or not the initial-state of the system can always be detected within a finite delay. In this paper, we provide new verification algorithms for checking these two properties. The idea is to use the reversed dynamic of the system to efficiently estimate the initial-state information. We show that our new results improve the complexity of existing verification algorithms for both properties. We illustrate our results by simple examples.**

I. Introduction

Discrete-event systems (DES) are dynamic systems with discrete state-spaces and event-driven dynamics. DES can be used to model both logic systems that are inherently event-driven and symbolic abstractions of continuous dynamic systems. In many problems, e.g., supervisory control and fault diagnosis, the state information of the system is usually crucial in order to make correct decisions. However, in many real world applications, we do not usually have perfect knowledge of the system due to measurement uncertainties. Therefore, state estimation and detection are important issues in the analysis and design of partially-observed DES.

In the context of DES, the problem of state estimation dates back to the study of the property of observability; see, e.g., [8], [13], [14]. In this problem, it is assumed that the system's behavior is only partially-known and we want to infer the system's "state" based on the imperfect information. The state estimation problem is closely related to many practical problems, including fault diagnosis problem [29], [31], [37], fault prognosis problem [24], [28] state disambiguation problem [16], [25], [33], [34], and information-flow security problem [4], [7], [36].

Recently, the state estimation of DES has been investigated in a more systematic manner in the context of detectability. The concept of detectability was first proposed by Shu and Lin in [22], where several different notions of detectability, e.g., strong (periodic) detectability and weak (periodic) detectability, are defined. Specifically, the authors of [22] considered DES modeled as finite-state automata with unobservable events. Then strong detectability captures whether or not we can always detect the current state of the system within a finite delay, while weak detectability captures whether or not we can detect the current state of the system via some path. Verification algorithms were also provided for different notions of detectability in [22]. In [18], a polynomial-time algorithm was provided for the verification of strong detectability. However, it has been shown more recently by [9], [10], [38] that verifying weak detectability is PSPACE-hard. Therefore, it is unlikely that a polynomial-time algorithm exists for the verification of weak detectability.

Since the seminal work of Shu and Lin, the concept of detectability has been studied more extensively and has been extended to more different settings. In [19], the concept of delayed detectability was proposed by allowing the usage of future observation for information smoothing. In [5], [6], [23], detectability was investigated in the stochastic setting by considering the transition probability of the system; corresponding stochastic notions of detectability capturing the probability of state detection were also provided. In [20], [30], the detectability enforcement problem was studied, where the goal is to design a maximally-permissive supervisor such that the controlled system is detectable. Recently, different new types of detectability are proposed in the literature for different detection requirements, e.g., K-detectability [3] and trajectory detectability [35]. Detectability has also been extended to different system models, including nondeterministic systems [40], fuzzy systems [12], Petri net systems [11], [39], networked systems [15] and modular systems [32].

In some applications, we may not interested in knowing the current-state of the system. Instead, we may be interested in detecting the *initial-state* of the system. To this end, the concept of I-detectability was introduced by [21]. Specifically, weak I-detectability captures whether or not the initial-state of the system can be detected via some path, while strong I-detectability captures whether or not the initial-state of the system can always be detected within a finite delay. Verification algorithms were also provided in [21] for both weak I-detectability and strong I-detectability. More recently, the initial-state detection problem has also been studied in the stochastic setting [27]. The usefulness of I-detectability has been supported by the work of [1], [2], where the authors apply I-detectability to smart home systems in order to tracking unknown inhabitants in a region.

In this paper, we revisit the verification of weak I-detectability and strong I-detectability as defined in [21]. The main contributions of this paper are as follows. First,

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we provide an improved approach for the verification of weak I-detectability. The complexity of previous algorithm proposed in [21] requires $O(|\Sigma|2^{|X|^2})$, where $|\Sigma|$ and $|X|$ are the number of events and number of states in the system, respectively. By using the reversed observer, we improve this complexity to $O(|\Sigma|2^{|X|})$. Also, we also provide an improved approach for the verification of strong I-detectability, whose complexity is $O(|\Sigma||X|^2)$ compared with complexity $O(|\Sigma||X|^4)$ of the previous algorithm. The idea is to construct a new structure called the reversed verifier that tracks the dynamic of the system reversely. This idea avoids using the state-augmenting procedure as used in [19] and improves the computational complexity.

The rest of this paper is organized as follows. Section II provides some necessary preliminaries. In Section III and IV, we study the verification of weak I-detectability and the verification of strong I-detectability, respectively. Finally, we conclude the paper in Section V.

II. Preliminaries

A. System Model

Let Σ be a finite set of events. A string $s = \sigma_1 \ldots \sigma_n$ is a finite sequence of events. We denote by $|s|$ the length of string s with $|\epsilon| = 0$, where ϵ is the empty string. We denote by Σ^* the set of all strings over Σ including the empty string ϵ. A language $L \subseteq \Sigma^*$ is a set of strings. We define $\Sigma_\epsilon := \Sigma \cup \{\epsilon\}$.

We consider a DES modeled as a non-deterministic finite-state automaton (NFA)

$$G = (X, \Sigma, \delta, X_0) \tag{1}$$

where

- X is the finite set of states;
- Σ is the finite set of events;
- $\delta : X \times \Sigma \to 2^X$ is the non-deterministic (partial) transition function;
- $X_0 \subseteq X$ is the set of initial-states.

For any $x, x' \in X, \sigma \in \Sigma$, $x' \in \delta(x, \sigma)$ implies that there exists a transition from x to x' labeled with σ. Function δ is also extended to $\delta : X \times \Sigma^* \to 2^X$ recursively as follows: for any $s \in \Sigma^*$ and $\sigma \in \Sigma$, we have $\delta(x, s\sigma) = \cup_{x' \in \delta(x,s)} \delta(x', \sigma)$. For each state x, we denote by $\mathcal{L}(G, x)$ the set of strings generated by G from x, i.e, $\mathcal{L}(G, x) = \{s \in \Sigma^* : \delta(x, s)!\}$, where "!" stands for "is defined". Therefore, $\mathcal{L}(G) = \cup_{x_0 \in X_0} \mathcal{L}(G, x_0)$ is the language generated by the system.

In many applications, the occurrence of event cannot be perfectly observed. To capture the imperfect observation, we assume that the event set is partitioned as

$$\Sigma = \Sigma_o \dot{\cup} \Sigma_{uo},$$

where Σ_o is the set of observable events and Σ_{uo} is the set of unobservable events. The natural projection $P : \Sigma^* \to \Sigma_o^*$ is defined recursively by: $\forall s \in \Sigma^*, \sigma \in \Sigma$

$$P(\epsilon) = \epsilon, \quad P(s\sigma) = \begin{cases} P(s)\sigma & \text{if } \sigma \in \Sigma_o \\ P(s) & \text{if } \sigma \in \Sigma_{uo} \end{cases}$$

The natural projection is also extended to $P : 2^{\Sigma^*} \to 2^{\Sigma_o^*}$ by: $\forall L \subseteq \Sigma^* : P(L) = \{P(s) \in \Sigma_o^* : s \in L\}$.

Finally, we make the following standard assumptions in the analysis of partially-observed DES:

A1 System G is deadlock-free, i.e., $\forall x \in X, \exists \sigma \in \Sigma : \delta(x, \sigma)!$;

A2 System G does not contain unobservable cycle, i.e., $\forall x \in X, \forall s \in \Sigma_{uo}^* : x \notin \delta(x, s)$.

B. Initial-State Detectability

In the initial-state detection problem, we assume that the system may initiated from states X_0; however, which specific state the system starts from is unknown. Therefore, the goal is to estimate and to detect the precise initial-state of the system based on the observation. To this end, for any $\alpha \in P(\mathcal{L}(G))$, we define

$$\hat{X}_0(\alpha) = \{x_0 \in X_0 : \exists s \in \mathcal{L}(G, x_0) \text{ s.t. } P(s) = \alpha\} \tag{2}$$

as the *initial-state estimate* of the system upon the occurrence of α.

In [21], two notions of initial-state detectability, weak I-detectability and strong I-detectability, are proposed in order to capture whether or not the initial-state of the system can be detected. These two definitions are reviewed as follows.

Definition 1: (Weak I-Detactability) A DES G is said to be *weakly I-detectable* w.r.t. Σ_o if there exists a string $\alpha \in P(\mathcal{L}(G))$ such that $|\hat{X}_0(\alpha)| = 1$.

Definition 2: (Strong I-Detactability) A DES G is said to be *strongly I-detectable* w.r.t. Σ_o if

$$(\exists n \in \mathbb{N})(\forall \alpha \in P(\mathcal{L}(G)))[|\alpha| \geq n \Rightarrow |\hat{X}_0(\alpha)| = 1] \tag{3}$$

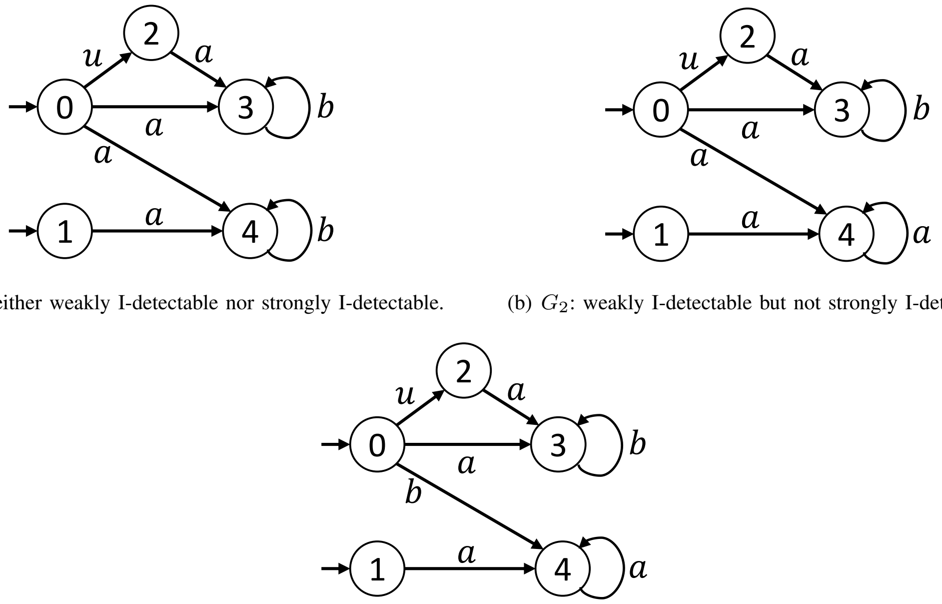
Intuitively, weak I-detectability requires that the initial-state of the systems can be determined for some trajectory, while strong I-detectability requires that the initial-state of the systems can be determined after a finite delay for any trajectory.

We illustrate weak I-detectability and strong I-detectability by the following example.

Example 1: Let us first consider system G_1 shown in Figure 1(a), where we have $\Sigma_o = \{a, b\}$ and input arrow without predecessor states presents that the corresponding state is an initial state, i.e., we have $X_0 = \{0, 1\}$ in G_1. Note that this system is neither weakly I-detectable nor strongly I-detectable, since the only possible observation is $abbb\ldots$ and we have $\hat{X}_0(ab^n) = \{0, 1\}$ for any $n \geq 0$.

However, system G_2 shown in Figure 1(b) is weakly I-detectable. For example, if we observe string ab, then we know for sure that the system must initially from state 0 and we have $\hat{X}_0(ab) = \{0\}$. However, this system is still not strongly I-detectable since $\hat{X}_0(a^n) = \{0, 1\}$ for any $n \geq 0$, i.e., we may not always be able to detect the initial-state of the system within a finite delay.

Finally, let us consider system G_3 shown in Figure 1(c). This system is both weakly I-detectable and strongly I-detectable. To see this, let us consider all possible observable



(a) G_1: neither weakly I-detectable nor strongly I-detectable. (b) G_2: weakly I-detectable but not strongly I-detectable.

(c) G_3: both weakly I-detectable and strongly I-detectable.

Fig. 1. Examples to illustrate detectability, where we have $\Sigma_o = \{a, b\}$ for G_1, G_2 and G_3.

strings with two events, i.e., strings ab, ba and aa. Then we have $\hat{X}_0(ab) = \hat{X}_0(ba) = \{0\}$ and $\hat{X}_0(aa) = \{1\}$. Therefore, we know that the initial-state of the system can always been detected within two steps. In the following sections, we will show formally how to verify weak I-detectability and strong I-detectability.

III. Verification of Weak I-Detectability

In [21], the authors have provided an algorithm for the verification of weak I-detectability. The idea is to first augment each state in G with the initial-state leading to this state, which leads to a new system with state-space $X \times X$. Then we can test weak I-detectability by constructing the observer automaton of the augmented system. The overall complexity of the algorithm in [21] is $O(|\Sigma| 2^{|X|^2})$. In this section, we provide a new approach for verifying weak I-detectability that improves the complexity of the algorithm in [21].

To this end, we define a new (deterministic) finite-state automaton called the *reversed observer*

$$Obs_R(G) = (Q, \Sigma_o, f, X)$$

where

- $Q \subseteq 2^X$ is the set of states;
- Σ_o is the set of observable events;
- $f : Q \times \Sigma_o \to Q$ is a (deterministic) transition function defined by: for any $q \in Q, \sigma \in \Sigma_o$, we have

  $$f(q, \sigma) = \{x \in X : \exists x' \in q, \exists w \in \Sigma_{uo}^* \text{ s.t. } x' \in \delta(x, w\sigma)\}$$

- $X \in Q$ is the unique initial state, which is the set of all states in G.

For the sake of simplicity, we only consider the reachable part of $Obs_R(G)$.

The idea of reversed observer was first presented in [26] for verifying opacity. Intuitively, it is similar to the standard observer automaton, but estimating states reversely. The following theorem shows how to use the reversed observer to verify weak I-detectability.

Theorem 1: System G is weakly I-detectable w.r.t. Σ_o if and only if there exists a state $q \in Q$ such that $|q \cap X_0| = 1$.

Proof: First, we claim that, for any $\alpha = \sigma_1 \dots \sigma_n \in P(\mathcal{L}(G))$, we have

$$f(X, \sigma_n \dots \sigma_1) = \{x \in X : \exists s \in \mathcal{L}(G, x) \text{ s.t. } P(s) = \alpha\} \tag{4}$$

We prove this claim by induction on n. When $n = 0$, i.e, $\alpha = \epsilon$, we know that $f(X, \epsilon) = X$ Therefore, the induction basis holds. Now we assume that Equation (4) holds for $|\alpha| = n$ and consider string $\sigma_1 \dots \sigma_{n+1} \in P(\mathcal{L}(G))$. Then we have

$$\begin{aligned}
& f(X, \sigma_{n+1}\sigma_n \dots \sigma_1) \\
=& f(f(X, \sigma_{n+1} \dots \sigma_2), \sigma_1) \\
=& \left\{ x \in X : \begin{array}{c} \exists x' \in f(X, \sigma_{n+1} \dots \sigma_2), \exists w \in \Sigma_{uo}^* \\ \text{s.t. } x' \in \delta(x, w\sigma_1) \end{array} \right\} \\
=& \left\{ x \in X : \begin{array}{c} \exists s \in \mathcal{L}(G, x'), \exists w \in \Sigma_{uo}^* \\ \text{s.t. } P(s) = \sigma_2 \dots \sigma_{n+1} \wedge x' \in \delta(x, w\sigma_1) \end{array} \right\} \\
=& \{x \in X : \exists s \in \mathcal{L}(G, x) \text{ s.t. } P(s) = \sigma_1 \dots \sigma_{n+1}\}
\end{aligned}$$

This proves our claim in Equation (4) and we further know that

$$\begin{aligned}
f(X, \sigma_n \dots \sigma_1) \cap X_0 =& \{x \in X_0 : \exists s \in \mathcal{L}(G, x) \text{ s.t. } P(s) = \alpha\} \\
=& \hat{X}_0(\alpha)
\end{aligned} \tag{5}$$

Therefore, by Equation (5), that G is weakly I-detectable, i.e., $\exists \alpha = \sigma_1 \dots \sigma_n \in P(\mathcal{L}(G))$ such that $|\hat{X}_0(\alpha)| = 1$, is equivalent to the existence of a state $f(X, \sigma_n \dots \sigma_1) \in Q$ such that $|f(X, \sigma_n \dots \sigma_1) \cap X_0| = |\hat{X}_0(\alpha)| = 1$. ∎

Remark 1: Theorem 1 provides a direct approach for the verification of weak I-detectability. Specifically, we just need to construct $Obs_R(G)$ and test whether or not a state q such that $|q \cap X_0| = 1$ can be reached. If so, then G is weakly I-detectability; otherwise, the system is not weakly I-detectable. The complexity of this approach only requires $O(|\Sigma_o| 2^{|X|})$. Recall that the complexity of the algorithm in [21] for checking weak I-detectability requires $O(|\Sigma_o| 2^{|X|^2})$. Therefore, our result provides an improved approach for checking this notion.

We illustrate how to use Theorem 1 to check weak I-detectability by the following example.

Example 2: Let us still consider systems G_1 and G_2 shown in Figures 1(a) and 1(b), respectively. Their reversed observers $Obs_R(G_1)$ and $Obs_R(G_2)$ are shown in Figures 2(a) and 2(b), respectively. For $Obs_R(G_1)$, we have $|\{0,1,2,3,4\} \cap X_0| = |\{0,1,2\} \cap X_0| = |\{0,1\}| = 2$ and $|\{3,4\} \cap X_0| = |\emptyset| = 0$. Therefore, by Theorem 1, we know that G_1 is not weakly I-detectable. However, for $Obs_R(G_2)$, we have $|\{0,2\} \cap X_0| = |\{0\}| = 1$. Therefore, by Theorem 1, we know that G_1 is weakly I-detectable.

IV. Verification of Strong I-Detectability

In this section, we investigate the verification of strong I-detectability. Note that, in [21], the authors have already provided an algorithm for verifying strong I-detectability by constructing a structure called the I-detector. The complexity of the algorithm proposed in [21] is $O(|\Sigma||X|^4)$. In this section, we show that this complexity can also be improved by providing a new algorithm.

Our approach for the verification of strong I-detectability is based on the construction of a new structure called the *reversed verifier*. Specifically, the reversed verifier for system G is a NFA

$$V_R(G) = (Q_V, \Sigma_V, f_V, Q_{0,V}) \tag{6}$$

where

- $Q_V = X \times X$ is the set of states;
- $\Sigma_V = (\Sigma_\epsilon \times \Sigma_\epsilon) \setminus \{(\epsilon, \epsilon)\}$ is the set of events;
- $Q_{0,V} = Q_V = X \times X$ is the set of initial states;
- The transition function $f_V : Q_V \times \Sigma_V \to 2^{Q_V}$ is defined as follows. For any $q = (x_1, x_2) \in Q_V, \sigma \in \Sigma$, the following transitions are defined:

  (a) If $\sigma \in \Sigma_o$, then

$$\begin{aligned} &f_V((x_1, x_2), (\sigma, \sigma)) \\ =&\{(x_1', x_2') \in Q_V : x_1 \in \delta(x_1', \sigma) \wedge x_2 \in \delta(x_2', \sigma)\} \end{aligned} \tag{7}$$

  (b) If $\sigma \in \Sigma_{uo}$, then

$$f_V((x_1, x_2), (\sigma, \epsilon)) = \{(x_1', x_2) \in Q_V : x_1 \in \delta(x_1', \sigma)\} \tag{8}$$

$$f_V((x_1, x_2), (\epsilon, \sigma)) = \{(x_1, x_2') \in Q_V : x_2 \in \delta(x_2', \sigma)\} \tag{9}$$

The reversed verifier is motivated by the standard verifier (or twin-machine) construction that has been widely used in the verification of partially-observed DES. Essentially, the structure tracks a pair of strings that have the same projection. The main difference between our construction and the standard verifier construction is that we track strings from all possible pairs of states and following the *reversed* dynamic of the original system.

The following results shows the main properties of the reversed verifier. First, we show that any pair of strings tracked by the reversed verifier have the same observation.

Proposition 1: For any sequence

$$(x_1^0, x_2^0) \xrightarrow[V]{(\sigma_1^1, \sigma_2^1)} (x_1^1, x_2^1) \xrightarrow[V]{(\sigma_1^2, \sigma_2^2)} \cdots \xrightarrow[V]{(\sigma_1^n, \sigma_2^n)} (x_1^n, x_2^n)$$

in $V_R(G)$, we have

(i) $\forall i = 1, 2 : \sigma_i^n \sigma_i^{n-1} \dots \sigma_i^1 \in \mathcal{L}(G, x_i^n)$; and
(ii) $P(\sigma_1^n \sigma_1^{n-1} \dots \sigma_1^1) = P(\sigma_2^n \sigma_2^{n-1} \dots \sigma_2^1)$.

Proof: Property (i) follows directly from the fact that for any $i = 1, 2$ and for any $k = 1, \dots, n$, we have $x_i^{k-1} \in \delta(x_i^k, \sigma_i^k)$. Note that this includes the case when $\sigma_i^k = \epsilon$. For property (ii), by the construction of $V_R(G)$, for each $k = 1, \dots, n$, there are only the following three cases:

- if $\sigma_1^k \in \Sigma_o$, then $P(\sigma_1^k) = P(\sigma_2^k) = \sigma_1^k = \sigma_2^k$
- if $\sigma_1^k \in \Sigma_{uo}$, then $\sigma_2^k = \epsilon$ and $P(\sigma_1^k) = P(\sigma_2^k) = \epsilon$;
- if $\sigma_1^k = \epsilon$, then $\sigma_2^k \in \Sigma_{uo}$ and $P(\sigma_1^k) = P(\sigma_2^k) = \epsilon$.

Therefore, we have $P(\sigma_1^n \dots \sigma_1^1) = P(\sigma_2^n \dots \sigma_2^1)$. ∎

Next, we show that, for any pair of strings that have the same observation, they will be included in the reversed verifier.

Proposition 2: For any two initial states $x_{0,1}, x_{0,2} \in X_0$ and two strings $s_1 \in \mathcal{L}(G, x_{0,1}), s_2 \in \mathcal{L}(G, x_{0,2})$ such that $P(s_1) = P(s_2)$, there must exist a sequence

$$(x_1^0, x_2^0) \xrightarrow[V]{(\sigma_1^1, \sigma_2^1)} (x_1^1, x_2^1) \xrightarrow[V]{(\sigma_1^2, \sigma_2^2)} \cdots \xrightarrow[V]{(\sigma_1^n, \sigma_2^n)} (x_1^n, x_2^n)$$

in $V_R(G)$ such that

1) $(x_1^n, x_2^n) = (x_{0,1}, x_{0,2})$; and
2) For any $i = 1, 2$, $s_i = \sigma_i^n \sigma_i^{n-1} \dots \sigma_i^1$.

Proof: We prove by induction on the length of $P(s_1) = P(s_2)$.

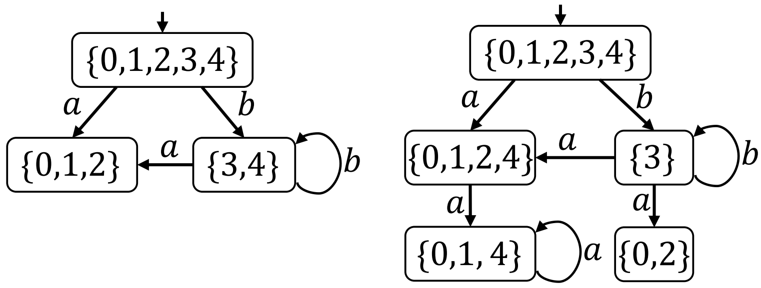
Induction Basis: When $|P(s_1)| = |P(s_2)| = 0$, for each $i = 1, 2$, we can write s_i in the form of $s_i = \sigma_i^1 \dots \sigma_i^{k_i}$, where $\sigma_i^j \in \Sigma_{uo}$. Therefore, for each $i = 1, 2$, there exist an initial state $\tilde{x}_i^0 \in X_0$ and a sequence

$$\tilde{x}_i^0 \xrightarrow{\sigma_i^1} \tilde{x}_i^1 \xrightarrow{\sigma_i^2} \cdots \xrightarrow{\sigma_i^{k_i}} \tilde{x}_1^{k_i} \tag{10}$$

Therefore, we can construct the following sequence in $V_R(G)$

$$\begin{aligned} (\tilde{x}_1^{k_1}, \tilde{x}_2^{k_2}) \xrightarrow[V]{(\epsilon, \sigma_2^{k_2})} \cdots \xrightarrow[V]{(\epsilon, \sigma_2^2)} (\tilde{x}_1^{k_1}, \tilde{x}_2^1) \xrightarrow[V]{(\epsilon, \sigma_2^1)} (\tilde{x}_1^{k_1}, \tilde{x}_2^0) \\ \xrightarrow[V]{(\sigma_1^{k_1}, \epsilon)} \cdots \xrightarrow[V]{(\sigma_1^2, \epsilon)} (\tilde{x}_1^1, \tilde{x}_2^0) \xrightarrow[V]{(\sigma_1^1, \epsilon)} (\tilde{x}_1^0, \tilde{x}_2^0) \end{aligned}$$

satisfying the conditions 1) and 2) in the proposition.



(a) $Obs_R(G_1)$. (b) $Obs_R(G_2)$.

Fig. 2. Examples of the reversed observer.

Induction Step: Let us assume that the proposition holds when $|P(s_1)| = |P(s_2)| = k$ and consider the case of $|P(s_1)| = |P(s_2)| = k+1$. In this case, for each $i = 1, 2$, we can write s_i in the form of $s_i = s_i' \sigma_i^0 \sigma_i^1 \dots \sigma_i^{k_i}$, where $\sigma_i^0 \in \Sigma_o$ and $\forall j \geq 1 : \sigma_i^j \in \Sigma_{uo}$. Note that, since $|P(s_1')| = |P(s_2')| = k$, by the induction hypothesis, there exists a sequence

$$(x_1^0, x_2^0) \xrightarrow[V]{(t_1,t_2)} (x_1^n, x_2^n), \text{ where } (t_1, t_2) \in \Sigma_V^*,$$

such that $\forall i = 1, 2 : (t_i)^R = s_i'$ and $(x_1^n, x_2^n) = (x_{0,1}, x_{0,2})$. Based on this sequence, we can further construct the following sequence in $V_R(G)$

$$(\tilde{x}_1^{k_1}, \tilde{x}_2^{k_2}) \xrightarrow[V]{(\epsilon,\sigma_2^{k_2})} \cdots \xrightarrow[V]{(\epsilon,\sigma_2^{2})} (\tilde{x}_1^{k_1}, \tilde{x}_2^{1}) \xrightarrow[V]{(\epsilon,\sigma_1^{1})} (\tilde{x}_1^{k_1}, \tilde{x}_2^{0})$$
$$\xrightarrow[V]{(\sigma_1^{k_1},\epsilon)} \cdots \xrightarrow[V]{(\sigma_1^{2},\epsilon)} (\tilde{x}_1^{1}, \tilde{x}_2^{0}) \xrightarrow[V]{(\sigma_1^{1},\epsilon)} (\tilde{x}_1^{0}, \tilde{x}_2^{0})$$
$$\xrightarrow[V]{(\sigma_1^{0},\sigma_2^{0})} (x_1^0, x_2^0) \xrightarrow[V]{(t_1,t_2)} (x_1^n, x_2^n)$$

satisfying the conditions 1) and 2) in the proposition. ■

In order to state our main result on the verification procedure, we first introduce some concepts. Let $V_R(G)$ be the reversed verifier. We denote by $Q_I \subseteq Q_V$ the set of states in which the first and the second components are two different initial states, i.e.,

$$Q_I := \{(x_1, x_2) \in Q_V : x_1, x_2 \in X_0 \text{ and } x_1 \neq x_2\} \quad (11)$$

A strongly connected component (SCC) in $V_R(G)$ is a maximal set of states $C \subseteq Q_V$ such that

$$\forall q, q' \in C, \exists s \in \Sigma_V^* : q' \in f_V(q, s) \quad (12)$$

A SCC $C \subseteq Q_V$ is said to be non-trivial if it contains at least one transition in it, i.e., it is not a single state without self-loops. For each SCC C, we denote by $\text{REACH}_V(C)$ the set of states reachable from C, i.e.,

$$\text{REACH}_V(C) = \{q \in Q_V : \exists q' \in C, s \in \Sigma_V^* \text{ s.t. } q \in f_V(q', s)\}.$$

The following result provides a necessary and sufficient condition for strong I-detectability.

Theorem 2: System G is strongly I-detectable w.r.t. Σ_o if and only if for any non-trivial SCC C, we have $\text{REACH}_V(C) \cap Q_I = \emptyset$.

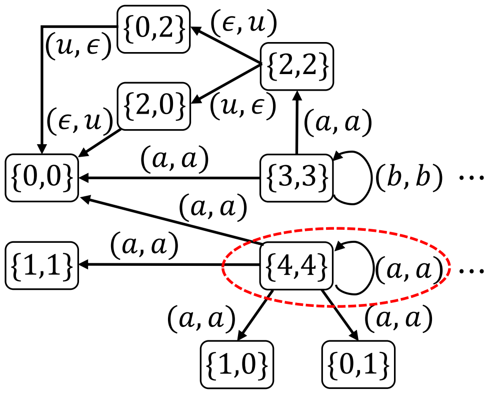


Fig. 3. Part of the reversed verifier $V_R(G_2)$ for system G_2 shown in Figure 1(b).

We illustrate how to verify strong I-detectability using Theorem 2.

Example 3: Let us consider system G_2 shown in Figure 1(b). We have discussed in Example 1 that this system is not strongly I-detectable; here, we verify this result by using Theorem 2. First, we construct the reversed verifier $V_R(G_2)$. For the sake of simplicity, we only depict part of $V_R(G_2)$ in Figure 3, which is sufficient for the purpose of disproving detectability. Then we see that state $(4, 4)$ is in a non-trivial SCC (since it has a self-loop event). Moreover, we have $(1, 0), (0, 1) \in \text{REACH}_V(\{(4, 4)\})$ and $(1, 0), (0, 1) \in Q_I$. Therefore, by Theorem 2, we can conclude that G_2 is not strongly I-detectable.

We conclude this section by discussing the complexity of verifying strong I-detectability using Theorem 2. In order to check strong I-detectability, first, we need to construct the reversed verifier, which contains at most $|X|^2$ states and $|\Sigma||X|^2$ transitions. Detecting all SCCs can be done in linear time in the size of the reserved verifier using the well-known Kosaraju's algorithm; see, e.g., [17]. Searching all reachable states from all non-trivial SCCs can also be done in linear time in the size of the reserved verifier using a simple depth-first search. Therefore, the overall complexity of verifying strong I-detectability using Theorem 2 is $O(|\Sigma||X|^2)$; this improves the previous algorithm in [21] whose complexity is $O(|\Sigma||X|^4)$.

V. Conclusion

In this paper, we revisited the verification of initial-state detectability in the context of partially-observed DES. First, we provided an improved approach for the verification of weak I-detectability by using the reversed observer. The complexity of the proposed algorithm is $O(|\Sigma|2^{|X|})$ compared with complexity $O(|\Sigma|2^{|X|^2})$ of the previous algorithm. Then, we provided a new approach for the verification of strong I-detectability by using the reversed verifier. The complexity of the proposed algorithm is $O(|\Sigma||X|^2)$ compared with complexity $O(|\Sigma||X|^4)$ of the previous algorithm. In the future, we plan to extend our results to the stochastic setting and to investigate how to improve the computational complexity for verifying stochastic I-detectability [27].

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