

Distributed H_∞ Filtering over Multiple-Channel Sensor Networks with Markovian Channel Switching and Time-varying Delays

Zhaojian Li, Xunyuan Yin, Xiang Yin, Yi Xie, Changhong Wang

Abstract—This paper is concerned with distributed H_∞ filtering for a class of continuous-time linear plants over sensor networks with multiple communication channels (MCCs). A practical framework is presented to optimize communication over MCCs with uncertain delays and switching characteristics. The channel switching is assumed to follow a continuous-time Markov chain and a Markov jump linear system (MJLS) is exploited to model the overall networked system. A class of Lyapunov-Krasovskii functionals are constructed to derive sufficient conditions on stochastic stability and H_∞ performance of the MJLS in terms of linear matrix inequalities. Furthermore, the filtering parameter design is presented by employing Projection Lemma and LMI techniques.

I. INTRODUCTION

Sensor network has become an increasingly attractive field over the past few decades and recent advancements in sensor hardware and communication technology have boosted its applications in a variety of areas, e.g., habitat monitoring [1], industrial automation [2], intelligent buildings [3] and health management [4], etc. A typical sensor network consists of a set of spatially distributed sensor nodes, collaborating with neighboring nodes to cooperatively monitor physical or environmental conditions. This networked sensing architecture brings about underlying advantages such as ability to cope with nodes failure and heterogeneity of nodes [5].

While traditional sensor networks employ one single communication channel for data transmission, modern sensor networks with multiple communication channels (MCCs) are emerging. MCCs can potentially alleviate transmission interference and thus improve the communication performance. The advantages of multiple channels over one single channel are emphasized in [6]–[8]. As a result, plenty of channel scheduling protocols have been investigated for multiple-channel sensor networks, see e.g., [8]–[10]. However, as far as the authors are concerned, the estimation problem for sensor networks with MCCs has not been studied yet, which motives us to place an emphasis on this topic.

In this paper, we consider a distributed H_∞ filtering problem for a linear plant over a sensor network with

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MCCs. See Figure 1. The treatment of H_∞ filtering over MCCS is different from existing studies in which only one single channel is considered [4], [11]. The channel switching phenomenon is supposed to be governed by a continuous-time Markov process and a Markov jump linear system (MJLS) is exploited to model the overall networked system. Lyapunov-Krasovskii functionals (LKF) are constructed to obtain distributed H_∞ filters that endure time-varying communication delays and random channel switching.

The remainder of this paper is organized as follows. In Section II, the distributed filtering problem for sensor network with switching MCCs is formulated and some background results are reviewed. Section III is devoted to the stability and H_∞ performance analysis. Explicit expressions for the distributed filters are designed in Section IV. Section V concludes the paper.

Notation: $L_2[0, \infty)$ is the space of square-integrable functions on $[0, \infty)$, and for $w(t) \in L_2[0, \infty)$, $\|w\|_2^2 = \int_0^\infty w(t)^T w(t) dt$. In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry. $diag_N\{x\}$ denotes the N -block diagonal matrix $diag\{x, \dots, x\}$. $diag_N^i\{x_i\}$ denotes the N -block-diagonal matrix with its i th block being x_i . Similarly, $col_N\{\cdot\}$ and $col_N^i\{\cdot\}$ denote column vectors with suitable blocks. $vec_N\{\cdot\}$ and $vec_N^i\{\cdot\}$ denote row vectors with suitable blocks. $Sym(A)$ is the shorthand notation for $A + A^T$. The Kronecker product is denoted by \otimes . For a symmetric matrix, $P > 0$ ($P \geq 0$) means that P is positive-(semi)definite.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider a target plant with the following continuous-time dynamics

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t), \\ z(t) = Lx(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector; $w(t) \in \mathbb{R}^{n_w}$ represents the disturbance signal belonging to $L_2[0, \infty)$ and $z(t) \in \mathbb{R}^{n_z}$ is the controlled output to be estimated. $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_w}$ and $L \in \mathbb{R}^{n_z \times n_x}$ are known constant matrices.

A sensor network with N nodes is deployed with a sensing topology represented by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of sensor node indices, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of paired sensor node edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the weighted adjacency matrix. An edge from node j to i is denoted by (i, j) . The elements that are associated with the edges in the adjacency matrix are positive, i.e., $a_{ij} > 0$, $\forall (i, j) \in \mathcal{E}$; otherwise, $a_{ij} = 0$.

In addition, self-loops are allowed, that is, (i, i) , $i \in \mathcal{V}$ are regarded as additional edges with $a_{ii} = 1$. Furthermore, for each node $i \in \mathcal{V}$, the set of its neighbors plus itself is represented by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.

For each sensor $i \in \mathcal{V}$, the measurement model is given by

$$y_i(t) = C_i x(t) + D_i w(t), \quad (2)$$

where $y_i(t) \in \mathbb{R}^{n_y}$ is the measurement output from sensor i ; $C_i \in \mathbb{R}^{n_y \times n_x}$ and $D_i \in \mathbb{R}^{n_y \times n_w}$ are known constant matrices.

A schematic diagram of data collection around sensor node i in a multi-channel sensor network is illustrated in Fig. 1. The sensor node i aggregates measurements from its affiliated sensor, sensor i , and all its underlying neighboring sensors $i_1, \dots, i_r \in \mathcal{N}_i$. The aggregated output of sensor node i is given by

$$\hat{y}_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij} y_j(t) \quad (3)$$

based on a prescribed topology \mathcal{G} .

We consider a framework that multiple communication channels (MCCs) are exploited for data transmission as in Fig. 1. Aggregated measurements $\hat{y}_i(t)$, $i = 1, \dots, N$ are sent to a network gateway, where the aggregated measurements from all sensor nodes are compressed and sent via the multiple-channel communication network. While M communication channels are available, only one of them is chosen and used at a time for signal transmissions. The channel selection is realized by exploiting a channel scheduler which chooses the best channel based on a prescribed protocol. A cooperative scheduler for the channels may use minimum time delay, minimum packet dropouts, or a minimum composite index of performance and cost as the channel selection protocol [7], [9], [10].

With a channel scheduler discussed above, the communication channel is switching with accordance to the prescribed protocol. Let c_t denote the channel switching signal taking values in a finite set $\mathcal{C} = \{1, 2, \dots, M\}$ identifying the channel currently being used. We assume the overall channel switching is governed by a continuous-time Markov process $\{c_t \in \mathcal{C}, t \geq 0\}$ with the following infinitesimal generator

$$\Lambda = [\lambda_{ij}], \quad i, j \in \mathcal{C}, \quad (4)$$

where $\lambda_{ij} \geq 0, \forall j \neq i, \lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$. Then the channel transition probability of channel i to channel j is approximated as

$$\Pr(c_{t+\Delta} = j | c_t = i) = \begin{cases} \lambda_{ij} \Delta + o(\Delta), & j \neq i, \\ 1 + \lambda_{ii} \Delta + o(\Delta), & j = i, \end{cases} \quad (5)$$

where $o(\Delta)$ denotes second or higher orders of Δ and $\lim_{\Delta \rightarrow 0} o(\Delta)/\Delta = 0$.

Time delays may occur during the signal transmissions. To be general, we assume different time-delay characteristics in the MCCs. The following assumptions are made about the communication delays.

Assumption 1: The communication delays if using channel c , $c \in \mathcal{C}$, denoted as $\tau_c(t)$, are time-varying and $\tau_c \leq \tau_c(t) \leq \bar{\tau}_c$, where τ_c and $\bar{\tau}_c$ are positive constant scalars representing the lower and upper bound of the communication delays of channel c , respectively. Also, we assume that $\dot{\tau}_c(t) \leq d < \infty, \forall c \in \mathcal{C}$, where d is a known bound. For notational simplicity, we may use τ_c to represent $\tau_c(t)$.

At the base station as in Figure 1, the transmitted data are received, decompressed and separated. Let $\tilde{y}_v^{c_t}(t)$ denote the received measurement aggregated by sensor node v , $v \in \mathcal{V}$, from the base station at time t if using channel c_t , we have

$$\tilde{y}_v^{c_t}(t) = \hat{y}_v(t - \tau_{c_t}(t)). \quad (6)$$

We consider full-order channel-dependent distributed filters for each of the sensors $v \in \mathcal{V}$ with the following form

$$\begin{aligned} \dot{\tilde{x}}_v(t) &= K_A^v(c_t) \tilde{x}_v(t) + K_B^v(c_t) \tilde{y}_v^{c_t}(t), \\ \tilde{z}_v(t) &= K_C^v(c_t) \tilde{x}_v(t), \end{aligned} \quad (7)$$

where $K_A^v(c_t) \in \mathbb{R}^{n_x \times n_x}$, $K_B^v(c_t) \in \mathbb{R}^{n_x \times n_y}$, $K_C^v(c_t) \in \mathbb{R}^{n_z \times n_x}$, $v \in \mathcal{V}$, $c_t \in \mathcal{C}$ are channel-dependent filter parameters to be designed.

Let $e_v(t) = z(t) - \tilde{z}_v(t)$ denote the filtering error of sensor node v , $v \in \mathcal{V}$. We further define the following matrices to facilitate subsequent analyses

$$\begin{aligned} \hat{A} &= \text{diag}_N\{A\}, \quad \hat{B} = \text{col}_N\{B\}, \\ \bar{K}_A(c_t) &= \text{diag}_N^v\{K_A^v(c_t)\}, \quad \bar{x}(t) = \text{col}_N\{x(t)\}, \\ \tilde{x} &= \text{col}_N^v\{\tilde{x}_v\}, \quad e(t) = \text{col}_N^v\{e_v(t)\}, \\ \bar{K}_B(c_t) &= \text{diag}_N^v\{K_B^v(c_t)\}, \quad \hat{C} = \text{vec}_N\{\text{col}_N^i\{C_i\}\}, \\ \hat{D} &= \text{col}_N^i\{D_i\}, \quad \eta(t) = \text{col}_2\{\bar{x}(t), \tilde{x}(t)\}, \\ \bar{L} &= \text{diag}_N\{L\}, \quad \bar{K}_C(c_t) = \text{diag}_N^v\{K_C^v(c_t)\}. \end{aligned}$$

By combining (1), (2), (3), (6) and (7), we have the following Markovian jump linear system (MJLS)

$$\begin{aligned} \dot{\eta}(t) &= \bar{A}(c_t) \eta(t) + \bar{A}_1(c_t) \eta(t - \tau_{c_t}(t)) + \bar{B}(c_t) w(t), \\ e(t) &= E(c_t) \eta(t), \\ \eta(s) &= \phi(s), \quad s \in [-\bar{\tau}, 0], \end{aligned} \quad (8)$$

where

$$\begin{aligned} \bar{A}(c_t) &= \begin{bmatrix} \hat{A} & 0_{N \cdot n_x \times N \cdot n_x} \\ 0_{N \cdot n_x \times N \cdot n_x} & \bar{K}_A(c_t) \end{bmatrix}, \\ \bar{A}_1(c_t) &= \begin{bmatrix} 0_{N \cdot n_x \times N \cdot n_x} & 0_{N \cdot n_x \times N \cdot n_x} \\ \frac{1}{N} \bar{K}_B(c_t) (\mathcal{A} \otimes I_{n_y}) \hat{C} & 0_{N \cdot n_x \times N \cdot n_x} \end{bmatrix}, \\ \bar{B}(c_t) &= \begin{bmatrix} \hat{B} \\ \bar{K}_B(c_t) (\mathcal{A} \otimes I_{n_y}) \hat{D} \end{bmatrix}, \quad E(c_t) = [\bar{L} \quad -\bar{K}_C(c_t)], \end{aligned}$$

and $\phi(\cdot) \in L_2[-\bar{\tau}, 0]$ is the initial condition with $\bar{\tau} = \max_{i \in \mathcal{C}} \tau_i$.

Definition 1: [12] System (8) with $w(t) \equiv 0$ is said to be stochastically stable (SS) if there exists a constant $T(c_0, \phi(\cdot)) > 0$, such that

$$\mathbb{E}[\|\eta\|^2 | (c_0, \phi(\cdot))] \leq T(c_0, \phi(\cdot)), \quad (9)$$

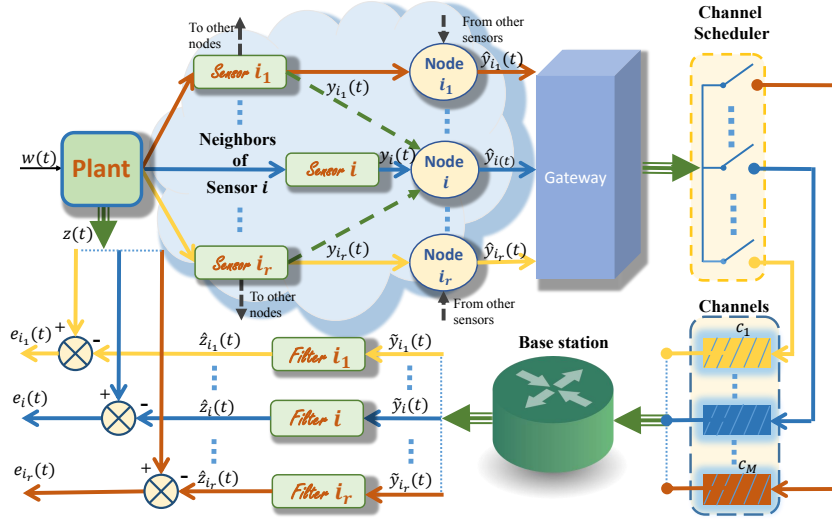


Fig. 1. A schematic diagram of sensor network with multiple channels around sensor node i .

where $\phi(s) \in L_2[-\bar{\tau}, 0]$ is the initial condition of system (8).

The desired distributed H_∞ filtering problem addressed in this paper can be formulated as follows: given the linear plant (1) over a multiple-channel sensor network as discussed and a prescribed level of noise attenuation $\gamma > 0$, determine linear filters in the form (7) such that the error system in (8) is stochastically stable and under zero initial conditions, the following average H_∞ performance is guaranteed.

$$\frac{1}{N} \sum_{v=1}^N \mathbb{E}[\|e_v(t)\|_2^2] \leq \gamma^2 \|w(t)\|_2^2. \quad (10)$$

Before ending this section, we introduce the following lemmas that will be used in the subsequent derivations and proofs.

Lemma 1: [13] If $f, g: [a, b] \rightarrow \mathbb{R}^n$ are similarly ordered, that is,

$$(f(x) - f(y))^T (g(x) - g(y)) \geq 0, \quad \forall x, y \in [a, b], \quad (11)$$

then,

$$\begin{aligned} & \frac{1}{b-a} \int_a^b f(x)g(x)dx \\ & \geq \left[\frac{1}{b-a} \int_a^b f(x)dx \right] \left[\frac{1}{b-a} \int_a^b g(x)dx \right]. \end{aligned} \quad (12)$$

Lemma 1 is a Chebyshev-like inequality.

Lemma 2: [14] Let $W = W^T \in \mathbb{R}^{n \times n}$, $U \in \mathbb{R}^{m \times n}$, and $V \in \mathbb{R}^{k \times n}$ be given matrices, and suppose that $\text{rank}(U) < n$ and $\text{rank}(V) < n$. The following LMI problem:

$$W + U^T \mathcal{X}^T V + V^T \mathcal{X} U < 0. \quad (13)$$

is solvable for \mathcal{X} if and only if

$$U_\perp^T W U_\perp < 0 \quad \text{if } V_\perp = 0, \quad U_\perp \neq 0,$$

$$V_\perp^T W V_\perp < 0 \quad \text{if } U_\perp = 0, \quad V_\perp \neq 0,$$

$$U_\perp^T W U_\perp < 0, \quad V_\perp^T W V_\perp < 0, \quad \text{if } V_\perp \neq 0, \quad U_\perp \neq 0,$$

where U_\perp and V_\perp represent the right null spaces of U and V , respectively. Lemma 2 is referred to as the Projection Lemma.

III. STABILITY AND PERFORMANCE ANALYSES

In this section, stability and H_∞ performance analysis for the networked error system (8) are presented and sufficient conditions on the existence of the desired filters are given in the following Theorem.

Theorem 1: Let $K_A^v(c)$, $K_B^v(c)$, $K_C^v(c)$, $v \in \mathcal{V}$, $c \in \mathcal{C}$, be given filter parameters. Then the error system (8), with $w(t) \equiv 0$, is stochastically stable and the average H_∞ performance in (10) is guaranteed under zero initial conditions if there exist positive-definite matrices $P(1), P(2), \dots, P(M)$, $Q_1, Q_2, Q_3, R_1, R_2 \in \mathbb{R}^{2N \cdot n_x \times 2N \cdot n_x}$ and matrix $\tilde{G} \in \mathbb{R}^{(10N \cdot n_x + n_w) \times 2N \cdot n_x}$ such that

$$\begin{aligned} & \begin{bmatrix} \mathcal{I}_1^T \mathcal{R} \mathcal{I}_1 & P(i)M_1 \\ * & M^T \mathcal{Q}(i) \mathcal{M} + \frac{1}{N} M_1^T E^T(i) E(i) M_1 \end{bmatrix} \\ & + \text{Sym} \left\{ \tilde{G} \begin{bmatrix} -I_{2N \cdot n_x} & \mathcal{H}(i) \end{bmatrix} \right\} < 0. \end{aligned} \quad (14)$$

for all $i \in \mathcal{C}$, where

$$\mathcal{H}(i) = [\bar{A}(i) \quad \bar{A}_1(i) \quad 0_{2N \cdot n_x \times 4N \cdot n_x} \quad \bar{B}(i)],$$

$$M_1 = [I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times (6N \cdot n_x + n_w)}],$$

$$M_2 = [0_{2N \cdot n_x \times 2N \cdot n_x} \quad I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times (4N \cdot n_x + n_w)}],$$

$$M_3 = [0_{2N \cdot n_x \times 4N \cdot n_x} \quad I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times (2N \cdot n_x + n_w)}],$$

$$M_4 = [0_{2N \cdot n_x \times 6N \cdot n_x} \quad I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times n_w}],$$

$$M_5 = [0_{n_w \times 8N \cdot n_x} \quad I_{n_w}], \quad \bar{\tau} = \max_{i \in \mathcal{C}} \bar{\tau}_i, \quad \underline{\tau} = \min_{i \in \mathcal{C}} \underline{\tau}_i,$$

$$\mathcal{M} = [M_1 \quad M_2 \quad M_3 \quad M_4 \quad M_5 \quad \mathcal{S}_1 \quad \mathcal{S}_2],$$

$$\mathcal{S}_1 = [I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times 2N \cdot n_x} \quad -I_{2N \cdot n_x}$$

$$0_{2N \cdot n_x \times (2N \cdot n_x + n_w)}], \quad \mathcal{R} = \underline{\tau}^2 R_1 + \bar{\tau}^2 R_2,$$

$$\mathcal{S}_2 = [I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times 4N \cdot n_x} \quad -I_{2N \cdot n_x} \quad 0_{2N \cdot n_x \times n_w}],$$

$$\mathcal{Q}(i) = \text{diag}\{\Xi(i), -(1-d)\mathcal{I}_1^\top Q_1 \mathcal{I}_1, -\mathcal{I}_1^\top Q_2 \mathcal{I}_1, -\mathcal{I}_1^\top Q_3 \mathcal{I}_1, \\ -\gamma^2 I_{n_w}, -\mathcal{I}_1^\top R_1 \mathcal{I}_1, -\mathcal{I}_1^\top R_2 \mathcal{I}_1\}, \quad \bar{\lambda} = \max_{i \in \mathcal{C}} \{\lambda_{ii}\},$$

$$\Xi(i) = \sum_{j=1}^N \lambda_{ij} P(j) + \mathcal{I}_1^\top [(1+\varrho)Q_1 + Q_2 + Q_3] \mathcal{I}_1,$$

$$\varrho = 1 + \bar{\lambda}(\bar{\tau} - \underline{\tau}), \quad \mathcal{I}_1 = [I_{N \cdot n_x} \quad 0_{N \cdot n_x \times N \cdot n_x}].$$

Proof: To prove the result, we construct a Markovian Lyapunov–Krasovskii functional (LKF) $V(\eta(t), c_t, t)$ similar in [15], [16] as follows

$$V(\eta(t), c_t, t) = \sum_{i=1}^4 V_i(\eta(t), c_t, t), \quad (15)$$

with

$$V_1(\eta(t), c_t, t) = \eta^\top(t) P(c_t) \eta(t),$$

$$V_2(\eta(t), c_t, t) = \int_{t-\tau_{c_t}(t)}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ + \int_{t-\underline{\tau}}^t \eta^\top(s) \mathcal{I}_1^\top Q_2 \mathcal{I}_1 \eta(s) ds + \int_{t-\bar{\tau}}^t \eta^\top(s) \mathcal{I}_1^\top Q_3 \mathcal{I}_1 \eta(s) ds \\ := V_{21} + V_{22} + V_{23},$$

$$V_3(\eta(t), c_t, t) = \int_{-\bar{\tau}}^{-\underline{\tau}} \int_{t+\theta}^t \eta^\top(s) \mathcal{I}_1^\top \bar{\lambda} Q_1 \mathcal{I}_1 \eta(s) ds d\theta,$$

$$V_4(\eta(t), c_t, t) = \underline{\tau} \int_{-\underline{\tau}}^0 \int_{t+\theta}^t \dot{\eta}^\top(s) \mathcal{I}_1^\top R_1 \mathcal{I}_1 \dot{\eta}(s) ds d\theta \\ + \bar{\tau} \int_{-\bar{\tau}}^0 \int_{t+\theta}^t \dot{\eta}^\top(s) \mathcal{I}_1^\top R_2 \mathcal{I}_1 \dot{\eta}(s) ds d\theta,$$

where \mathcal{I}_1 and $\bar{\lambda}$ are defined in the statement of Theorem 1.

Let \mathcal{D} be the weak infinitesimal generator of a random process $\{\eta(t), c_t\}$ and define

$$\mathcal{D}V(\eta(t), c_t, t) \\ = \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} \left\{ \mathbb{E}[V(\eta(t+\Delta), c_{t+\Delta}, t+\Delta)] - V(\eta(t), c_t, t) \right\}.$$

Applying the weak infinitesimal generator to the LKF (15), we have

$$\mathcal{D}V_1(\eta(t), c_t, t) = 2\eta^\top(t) P(c_t) \dot{\eta}(t) + \eta^\top(t) \sum_{j=1}^N \lambda_{c_t j} P(j) \eta(t). \quad (16)$$

We next compute the infinitesimal of $V_2(x(t), c_t, t)$.

$$\mathcal{D}V_{21}(\eta(t), c_t, t) \\ = \sum_{j \in \mathcal{C}} \lambda_{c_t j} \int_{t-\tau_j}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds + \eta^\top(t) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(t) \\ - (1 - \dot{\tau}_{c_t}) \eta^\top(t - \tau_{c_t}) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(t - \tau_{c_t}). \quad (17)$$

Simple computation yields

$$\mathcal{D}V_{22}(\eta(t), c_t, t) + \mathcal{D}V_{23}(\eta(t), c_t, t) \\ = \eta^\top(t) \mathcal{I}_1^\top (Q_2 + Q_3) \mathcal{I}_1 \eta(t) - \eta^\top(t - \underline{\tau}) \mathcal{I}_1^\top Q_2 \mathcal{I}_1 \eta(t - \underline{\tau}) \\ - \eta^\top(t - \bar{\tau}) \mathcal{I}_1^\top Q_3 \mathcal{I}_1 \eta(t - \bar{\tau}). \quad (18)$$

From (17), (18), and Assumption 1, it follows that

$$\mathcal{D}V_2(\eta(t), c_t, t) \\ \leq \eta^\top(t) \mathcal{I}_1^\top \sum_{i=1}^3 Q_i \mathcal{I}_1 \eta(t) \\ + \sum_{j \in \mathcal{C}} \lambda_{c_t j} \int_{t-\tau_j}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ - (1-d) \eta^\top(t - \tau_{c_t}) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(t - \tau_{c_t}) \\ - \eta^\top(t - \underline{\tau}) \mathcal{I}_1^\top Q_2 \mathcal{I}_1 \eta(t - \underline{\tau}) - \eta^\top(t - \bar{\tau}) \mathcal{I}_1^\top Q_3 \mathcal{I}_1 \eta(t - \bar{\tau}). \quad (19)$$

Direct computations yield

$$\mathcal{D}V_3(\eta(t), c_t, t) \\ = \bar{\lambda}(\bar{\tau} - \underline{\tau}) \eta^\top(t) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(t) \\ - \bar{\lambda} \int_{t-\bar{\tau}}^{t-\underline{\tau}} \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \quad (20)$$

and

$$\mathcal{D}V_4(\eta(t), c_t, t) \\ = \underline{\tau}^2 \dot{\eta}^\top(t) \mathcal{I}_1^\top R_1 \mathcal{I}_1 \dot{\eta}(t) + \bar{\tau}^2 \dot{\eta}^\top(t) \mathcal{I}_1^\top R_2 \mathcal{I}_1 \dot{\eta}(t) \\ - \underline{\tau} \int_{t-\underline{\tau}}^t \dot{\eta}^\top(s) \mathcal{I}_1^\top R_1 \mathcal{I}_1 \dot{\eta}(s) ds \\ - \bar{\tau} \int_{t-\bar{\tau}}^t \dot{\eta}^\top(s) \mathcal{I}_1^\top R_2 \mathcal{I}_1 \dot{\eta}(s) ds \\ \leq \underline{\tau}^2 \dot{\eta}^\top(t) \mathcal{I}_1^\top R_1 \mathcal{I}_1 \dot{\eta}(t) + \bar{\tau}^2 \dot{\eta}^\top(t) \mathcal{I}_1^\top R_2 \mathcal{I}_1 \dot{\eta}(t) \\ - [\eta(t) - \eta(t - \underline{\tau})]^\top \mathcal{I}_1^\top R_1 \mathcal{I}_1 [\eta(t) - \eta(t - \underline{\tau})] \\ - [\eta(t) - \eta(t - \bar{\tau})]^\top \mathcal{I}_1^\top R_2 \mathcal{I}_1 [\eta(t) - \eta(t - \bar{\tau})], \quad (21)$$

where the last inequality follows from Lemma 1.

Also in (19),

$$\sum_{j=1}^N \lambda_{c_t j} \int_{t-\tau_j}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ = \sum_{j \neq c_t} \lambda_{c_t j} \int_{t-\tau_j}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ + \lambda_{c_t c_t} \int_{t-\tau_{c_t}}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ \leq \sum_{j \neq c_t} \lambda_{c_t j} \int_{t-\bar{\tau}}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ + \lambda_{c_t c_t} \int_{t-\underline{\tau}}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds.$$

Noting that $\sum_{j \neq c_t} \lambda_{c_t j} = -\lambda_{c_t c_t}$, it follows that

$$\begin{aligned} & \sum_{j=1}^N \lambda_{c_t j} \int_{t-\tau_j}^t \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ & \leq -\lambda_{c_t c_t} \int_{t-\bar{\tau}}^{t-\underline{\tau}} \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds \\ & \leq \bar{\lambda} \int_{t-\bar{\tau}}^{t-\underline{\tau}} \eta^\top(s) \mathcal{I}_1^\top Q_1 \mathcal{I}_1 \eta(s) ds. \end{aligned} \quad (22)$$

Suppose $c_t = i \in \mathcal{C}$ and define the following augmented state

$$\zeta_i(t) = [\eta^\top(t) \quad \eta^\top(t - \tau_i) \quad \eta^\top(t - \underline{\tau}) \quad \eta^\top(t - \bar{\tau}) \quad w^\top(t)]^\top.$$

Then it is straightforward to check that

$$\begin{aligned} \eta(t) &= M_1 \zeta_i(t), & \eta(t - \tau_i) &= M_2 \zeta_i(t), \\ \eta(t - \underline{\tau}) &= M_3 \zeta_i(t), & \eta(t - \bar{\tau}) &= M_4 \zeta_i(t), \\ w(t) &= M_5 \zeta_i(t), & \dot{\eta}(t) &= \mathcal{H}(i) \zeta_i(t), \\ \eta(t) - \eta(t - \underline{\tau}) &= \mathcal{S}_1 \zeta_i(t), & \eta(t) - \eta(t - \bar{\tau}) &= \mathcal{S}_2 \zeta_i(t), \end{aligned} \quad (23)$$

with $M_1, \dots, M_5, \mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{H}(i)$ defined in the statement of Theorem 1.

As a result, it follows from (16), (19)-(23) that

$$\begin{aligned} DV(\eta(t), c_t, t) + \frac{1}{N} e^\top(t) e(t) - \gamma^2 w^\top(t) w(t) \\ \leq \zeta_i^\top(t) \Pi(i) \zeta_i(t) \end{aligned}$$

with

$$\begin{aligned} \Pi(i) &= 2M_1^\top P(i) \mathcal{H}(i) + M_1^\top \sum_{j=1}^N \lambda_{ij} P(j) M_1 \\ &+ M_1^\top \mathcal{I}_1^\top \sum_{i=1}^3 Q_i \mathcal{I}_1 M_1 - (1-d) M_2^\top \mathcal{I}_1^\top Q_1 \mathcal{I}_1 M_2 \\ &- M_3^\top \mathcal{I}_1^\top Q_2 \mathcal{I}_1 M_3 - M_4^\top \mathcal{I}_1^\top Q_3 \mathcal{I}_1 M_4 + M_1^\top \mathcal{I}_1^\top \varrho Q_1 \mathcal{I}_1 M_1 \\ &+ \mathcal{H}^\top(i) \mathcal{I}_1^\top \mathcal{R} \mathcal{I}_1 \mathcal{H}(i) - \mathcal{S}_1^\top \mathcal{I}_1^\top R_1 \mathcal{I}_1 \mathcal{S}_1 - \mathcal{S}_2^\top \mathcal{I}_1^\top R_2 \mathcal{I}_1 \mathcal{S}_2 \\ &+ \frac{1}{N} M_1^\top E^\top(i) E(i) M_1 - \gamma^2 M_5^\top M_5 \\ &= \mathcal{H}^\top(i) \mathcal{I}_1^\top \mathcal{R} \mathcal{I}_1 \mathcal{H}(i) + 2M_1^\top P(i) \mathcal{H}(i) \\ &+ \mathcal{M}^\top \mathcal{Q}(i) \mathcal{M} + \frac{1}{N} M_1^\top E^\top(i) E(i) M_1 \\ &= U_\perp^\top W U_\perp, \end{aligned} \quad (24)$$

where $U_\perp = \begin{bmatrix} \mathcal{H}(i) \\ I_{8N \cdot n_x + n_w} \end{bmatrix}$,

$$W = \begin{bmatrix} \mathcal{I}_1^\top \mathcal{R} \mathcal{I}_1 & P(i) M_1 \\ * & \mathcal{M}^\top \mathcal{Q}(i) \mathcal{M} + \frac{1}{N} M_1^\top E^\top(i) E(i) M_1 \end{bmatrix}.$$

By assigning $\tilde{G} \rightarrow X$ and $I_{10N \cdot n_x + n_w} \rightarrow V$, $0_{(10N \cdot n_x + n_w) \times (10N \cdot n_x + n_w)} \rightarrow V_\perp$ and applying Lemma 2, it can be shown that $\Pi(i) < 0$ is equivalent to (14) in Theorem 1, which implies that

$$DV(\eta(t), c_t, t) + \frac{1}{N} e^\top(t) e(t) - \gamma^2 w^\top(t) w(t) < 0.$$

Thus it follows from Theorem 10.2 in [12] that the filtering error system in (8) is stochastically stable and the average H_∞ filtering performance in (10) is obtained. This completes the proof of Theorem 1. \blacksquare

Remark 1: The Projection Lemma is applied by introducing a slack matrix \tilde{G} such that the filter parameters and Lyapunov matrices are decoupled. We note that this decoupling procedure provides a simpler method for analysis and design compared with many traditional methods [12], [17], [18].

IV. FILTER DESIGN

In this section, we solve the H_∞ filtering design problem for the sensor network system represented in (1)-(3), that is, finding filtering gains $K_A^v(c)$, $K_B^v(c)$ and $K_C^v(c)$, $v \in \mathcal{V}$, $c \in \mathcal{C}$, in forms of (7) such that the error system (8) is stochastically stable with a guaranteed average H_∞ bound. Sufficient conditions for the existence of such filters and parameter designs are given in the following Theorem.

Theorem 2: Let γ be a given constant representing the desired attenuation level. For the sensor network system described in (1)-(3), there exist distributed filters in forms of (7) such that the filtering error system (8) is stochastically stable with a guaranteed H_∞ performance index γ , if there exist positive-definite matrices $P(i) = \begin{bmatrix} P_1(i) & P_2(i) \\ * & P_3(i) \end{bmatrix} \in \mathbb{R}^{2N \cdot n_x \times 2N \cdot n_x}$, $i = 1, 2, \dots, M$, $Q_1, Q_2, Q_3, R_1, R_2 \in \mathbb{R}^{2N \cdot n_x \times 2N \cdot n_x}$, $G_2 = \text{diag}_N\{\mathcal{G}_v\} \in \mathbb{R}^{N \cdot n_x \times N \cdot n_x}$, positive constants $\varrho_1, \varrho_2, \varrho_3$, matrices $G_1, G_3 \in \mathbb{R}^{N \cdot n_x \times N \cdot n_x}$, $\tilde{K}_A^v(i) \in \mathbb{R}^{N \cdot n_x \times N \cdot n_x}$, $\tilde{K}_B^v(i) \in \mathbb{R}^{N \cdot n_x \times N \cdot n_w}$, $K_C(i) \in \mathbb{R}^{N \cdot n_x \times N \cdot n_z}$, $v = 1, 2, \dots, N$, $i = 1, 2, \dots, M$, such that

$$\begin{bmatrix} \Gamma_1(i) & \Gamma_2(i) \\ * & -N \cdot I_{2N \cdot n_x} \end{bmatrix} < 0 \quad (25)$$

for all $i \in \mathcal{C}$, where

$$\begin{aligned} \Gamma_1(i) &= \begin{bmatrix} \mathcal{I}_1^\top \mathcal{R} \mathcal{I}_1 & P(i) M_1 \\ * & \mathcal{M}^\top \mathcal{Q}(i) \mathcal{M} \end{bmatrix} \\ &+ \text{Sym} \left\{ \begin{bmatrix} -\pi \otimes G & \pi \otimes \hat{\mathcal{H}}(i) \\ 0_{n_w \times 2N \cdot n_x} & 0_{n_w \times (8N \cdot n_x + n_w)} \end{bmatrix} \right\}, \end{aligned}$$

$$\Gamma_2(i) = \begin{bmatrix} 0_{2N \cdot n_x \times 2N \cdot n_z} \\ M_1^\top E^\top(i) \end{bmatrix}, \quad \pi = [1 \quad \varrho_1 \quad \varrho_2 \quad \varrho_3]^\top,$$

$$\begin{aligned} \hat{\mathcal{H}}(i) &= \begin{bmatrix} G_1 \text{diag}_N\{A\} & \text{diag}_N^v\{\tilde{K}_A^v(i)\} & \frac{1}{N} \text{diag}_N^v\{\tilde{K}_B^v(i)\} \\ G_3 \text{diag}_N\{A\} & \text{diag}_N^v\{\tilde{K}_A^v(i)\} & \frac{1}{N} \text{diag}_N^v\{\tilde{K}_B^v(i)\} \\ 0 & G_1 \text{col}_N\{B\} + \text{diag}_N^v\{\tilde{K}^v(i)\} (A \otimes I_{n_y}) \hat{D} \\ 0 & G_3 \text{col}_N\{B\} + \text{diag}_N^v\{\tilde{K}^v(i)\} (A \otimes I_{n_y}) \hat{D} \end{bmatrix}, \end{aligned}$$

and $\mathcal{R}, \mathcal{I}_1, M_1, \mathcal{M}$ and $E(i)$ are defined the same as in the statement of Theorem 1. Furthermore, if (25) is feasible, then the filter parameters in (7) are given as

$$K_A^v(c) = \mathcal{G}_v^{-1} \tilde{K}_A^v(c), \quad K_B^v(c) = \mathcal{G}_v^{-1} \tilde{K}_B^v(c), \quad (26)$$

for all $c \in \mathcal{C}$, $v \in \mathcal{V}$.

Proof: According to Theorem 1, the filtering error system in (8) is stochastically stable and satisfies an average

H_∞ performance as in (10) if (14) is satisfied. For the filter synthesis procedure, we first specify the slack matrix \tilde{G} as

$$\tilde{G} = \begin{bmatrix} G^T & \varrho_1 G^T & \varrho_2 G^T & \varrho_3 G^T & 0_{2N \cdot n_x \times n_w} \end{bmatrix}^T, \quad (27)$$

$$G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix},$$

where $G_1, \dots, G_4 \in \mathbb{R}^{N \cdot n_x \times N \cdot n_x}$, and ϱ_1, ϱ_2 , and ϱ_3 are scalar parameters to be searched.

Then by virtue of congruence transformation similar to [17], [19], [20], we perform a transformation to G by $\text{diag}\{I_{N \cdot n_x}, G_2 G_4^{-1}\}$ for matrix inequalities linearization, which yields

$$\begin{bmatrix} I_{N \cdot n_x} & 0 \\ 0 & G_2 G_4^{-1} \end{bmatrix} \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix} \begin{bmatrix} I_{N \cdot n_x} & 0 \\ 0 & G_4^{-1} G_2^T \end{bmatrix}$$

$$= \begin{bmatrix} G_1 & G_2 G_4^{-T} G_2^T \\ G_2 G_4^T G_3 & G_2 G_4^{-T} G_2^T \end{bmatrix}.$$

As a result, we can directly specify the matrices, without loss of generality, as

$$G = \begin{bmatrix} G_1 & G_2 \\ G_3 & G_2 \end{bmatrix}. \quad (28)$$

We note that with G in the form of (28) and specify $G_2 = \text{diag}\{G_i\}$, the matrices $K_A^v(c)$ and $K_B^v(c)$, $v \in \mathcal{V}$, $c \in \mathcal{C}$ can be absorbed by introducing

$$\tilde{K}_A^v(c) = \mathcal{G}_v K_A^v(c), \quad \tilde{K}_B^v(c) = \mathcal{G}_v K_B^v(c).$$

By using Schur complement and substituting \tilde{G} with the specifications in (27) and (28), it can be shown that (14) and (25) are equivalent, which guarantees the stability and H_∞ performance. Note that $G_2 > 0$ requires $\mathcal{G}_v > 0$ for all $v \in \mathcal{V}$. As a result, the filter gains can be constructed using (26). This completes the proof of Theorem 2. ■

Remark 2: Note that in Theorem 2, three tuning parameters ϱ_1, ϱ_2 , and ϱ_3 are introduced to add extra freedom of the solution space, which is expected to lead to less conservative results. When these parameters are given, (25) is a strict LMI that can be solved with standard LMI solvers [21]. Non-gradient based optimizers such as MATLAB *fminsearch* can be exploited for tuning ϱ_1, ϱ_2 , and ϱ_3 . See [20], [22] for similar use of *fminsearch* in robust control applications.

V. CONCLUSIONS AND FUTURE WORK

The problem of distributed H_∞ filtering for a class of linear plants over a multiple-channel sensor network has been investigated. A Markov jump linear system has been employed to model the overall networked filtering system with Markovian channel switching. Channel-dependent time-varying communication delays have been considered. The Lyapunov-Krasovskii functional approach and LMI techniques have been exploited to establish the existence of the desired filters and to further derive the filter parameters. A real world application will be pursued as a case study in the future.

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