2021 60th IEEE Conference on Decision and Control (CDC)
December 13-15, 2021. Austin, Texas

Distributed Sensing and Information Transmission of Discrete-Event Systems with Edge Sensors

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***Abstract*— In this paper, we investigate the problem of distributed sensing and information transmission in partially observed discrete-event systems, where the sensing and information transmission are complicated by a set of edge sensors. Each edge sensor *selectively transmits* its observable events, according to *information transmission policies*, to a central site for the purpose of decision making. In this paper, we consider a general class of decision-making requirement at the central site called the *distinguishability*. Then we investigate both the verification and synthesis problems. For the verification problem, two different approaches, one based on the observer and the other based on the verifier, are proposed to check whether or not a given set of sensor transmission policies fulfills the distinguishability requirement at the central site. For the synthesis problem, we also develop an effective algorithm to design an observer-based optimal information transmission policy for each edge sensor such that they are verified to be distinguishable.**

I. INTRODUCTION

A. Motivations

Sensing and information transmission (SIT) problems have found increasing attentions in discrete-event systems where the systems make decisions under limited sensor capacities. Based on different SIT policies, several properties have been well studied in the DES literature to capture different requirements imposed on the system. Examples include fault diagnosis [1], [2], [3], [4] and opacity [5], [6], [7], [8], [9], [10]. There are two problems immediately: how to verify that a system satisfies (at least) one of those properties under a given SIT policy and how to synthesize a SIT policy to enforce the given property provably. In this paper, we investigate verification and synthesis problems of distributed SIT in the context of partially-observed DES with edge sensors.

The SIT problems in DES has been widely studied in the DES literature. For instance, in [11], [12] approaches are proposed to find a language-based minimal SIT policy for each agent such that the agents can always make a correct global decision as a team. Two algorithms that compute minimal SIT policies are proposed in [13] to verify several properties such as observability, diagnosability, detectability, and feasibility. The authors in [14] study optimal SIT problem in DES by placing a minimum number of sensors while

This work was supported by the National Key Research and Development Program of China (2018AAA0101700) and the National Natural Science Foundation of China (62061136004, 61803259, 61833012) and Shanghai Jiao Tong University Scientific and Technological Innovation Funds. The authors are with the Department of Automation and Key Laboratory of System Control and Information Processing, Shanghai Jiao Tong University, Shanghai 200240, China. E-mail: {yingyingliu611,yinxiang,syli}@sjtu.edu.cn.

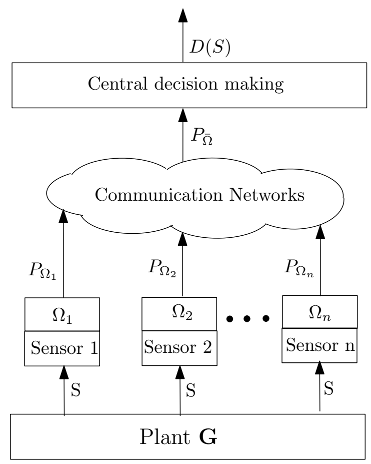

Fig. 1. Architecture of distributed dynamic sensor observations for centralized decision-making with n sensors and m agents, where Ω_i denotes the event transmission policy of sensor i and P_{Ω_i} denotes the information mapping under policy Ω_i (see strict definitions in Section II).

maintaining structural observability. A discrete-event system approach is given in [15] to investigate the information release problem, ensuring the opacity of private information while releasing the maximum information to the public.

B. Our Contribution

In this paper, we consider the distributed SIT problems in the context of partially-observed DES. As shown in Fig. 1, a new architecture is proposed to tackle the problem of distributed sensing and information transmission for the purpose of centralized decision making. In this architecture, each edge sensor *transmits* its observable events to a central site for decision making based on its SIT policy, called *information transmission policy*, which controls the observability property of events and decides the current observable event can be transmitted to the central site or not. The central site makes control decisions for each agent by collecting the information transmitted from the edge sensors. Note that, to reduce the cost of sensor readings (for reasons of bandwidth, energy, or security), the edge sensors have no communication with each other and do not receive any information from the central site.

The contributions of this paper are threefold. Firstly, we give a novel architecture based on the distributed SIT, where the edge sensors transmit the observable events to a central site for decision making based on their information transmission policies. Secondly, we propose two different approaches to verify under the given information trans-

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mission policy whether or not the system satisfies a class of properties, called *state disambiguation*, that is able to distinguish between the states of the system under dynamic observations and can be applied, but not restricted to, safety and opacity. One approach is based on building an observer from the central site. Another one is based on a verifier structure. Finally, we propose an algorithm to synthesize the optimal information transmission policies for edge sensors while ensuring the state distinguishability.

C. Related Works

The verification and synthesis problems under *static observations* have been widely studied in the DES literature by fixing observable events, where a given DES-theoretic property is satisfied [16], [17], [18], [19]. In [16], a methodology is proposed to obtain a set of observable events by exploiting the structure of the diagnoser automaton, which ensures language diagnosability of discrete-event systems. Instead of using partial diagnosers, test diagnosers, and other new constructs to achieve diagnosability as given in [16], in this paper we directly employ a centralized decision-maker to solve a kind of decision-making problems which includes the diagnosability problem. Authors in [18] synthesize an optimal set of sensors that can provide sufficient yet minimal events needed to accomplish the task at hand, such as that of control or estimation. The case of given sensors with fixed observation capability does not considered in [18]. By contrast, in this paper we discuss how to verify whether the given observation information is sufficient when the observation capability of sensors are given.

Dynamic observations [20], [21], [13], [22] also received a lot of attention in the context of centralized [23], [24], [25], [26], [27], decentralized [11], and distributed architectures[28], where sensors can be turned ON/OFF dynamically. For example, in the centralized architecture a new hierarchical framework is proposed in [27] to tackle decentralized diagnosis, where different distributed SIT policies together with appropriate rules are developed. Instead of constructing a diagnoser composed of a state estimator and a failure decision-maker as given in [27], in this paper we directly employ a centralized decision-maker to complete the state estimation and decision-making. In the decentralized architecture, many different property verification problems have already been studied. For instance, in [29] a SIT policy is defined by the diagnostic information generated at the distributed sites, the communication rules used by the distributed sites, and the coordinator's decision rule to address the problem of the failure diagnosis. In this paper, we generalize the coordinated decentralized architecture proposed in [29], so that local sites no longer need to make local diagnosis, but use a central decision-maker to judge not only diagnosis problem, but also a kind of decision-making problems. In the context of distributed architecture, diagnosis and communication problems are studied in [28], where local sites are required to communicate to perform some specified monitoring and control tasks. By construct, in this paper the local sites have no communication with each other and do not receive any information from the central site to reduce the cost of communications.

II. Distributed Information Transmission Architecture

A. System Model

We consider a DES modeled by a deterministic finite-state automaton (DFA)

$$\mathbf{G} = (Q, \Sigma, \delta, q_0, Q_m),$$

where Q is a finite set of states, Σ is a finite set of events, $\delta\colon Q \times \Sigma \to Q$ is a (partial) transition function, $q_0 \in Q$ is the initial state and $Q_m \subseteq Q$ is a set of marked states. In the usual way, δ can be extended to $\delta\colon Q \times \Sigma^* \to Q$, where Σ^* is the set of all finite-length *strings*, including the empty string ε. The *generated behavior* of $\mathbf{G}$ is language $L(\mathbf{G}) = \{s \in \Sigma^* : \delta(q_0, s)!\}$, where $\delta(q_0, s)!$ means that $\delta(q_0, s)$ is defined, and the *marked behavior* of $\mathbf{G}$ is the language $L_m(\mathbf{G}) = \{s \in L(\mathbf{G}) : \delta(q_0, s) \in Q_m\} \subseteq L(\mathbf{G})$.

A string $s_1 \in \Sigma^*$ is a *prefix* of $s \in \Sigma^*$, written as $s_1 \leq s$, if there is a string $s_2 \in \Sigma^*$ such that $s_1 s_2 = s$. The length of a string s is denoted by $|s|$. The *prefix closure* of a language L is the set $\overline{L} = \{s \in \Sigma^* : \exists t \in L \text{ s.t. } s \leq t\}$. For a natural number n, let $[1, n] = \{1, \ldots, n\}$ denote the set of all natural numbers from 1 to n.

B. Information Transmission Policy

We consider the scenario where system $\mathbf{G}$ is equipped with a set of *edge sensors* $\{S_1, S_2, \ldots, S_n\}$ that monitors the global system distributively. We denote by $\mathcal{I} = \{1, \ldots, n\}$ the index set. For each edge sensor $S_i, i \in \mathcal{I}$, we assume that it can only observe a set of locally observable event $\Sigma_{o,i} \subseteq \Sigma$. For each $i \in \mathcal{I}$, we denote by $P_i : \Sigma^* \to \Sigma_{o,i}^*$ the standard natural projection from Σ to $\Sigma_{o,i}$. However, upon the occurrence of a locally observable event $\sigma \in \Sigma_{o,i}$, an edge sensor does not necessarily need to transmit this observation to the central decision-maker. Note that here we consider a generic central decision-maker and it can be, e.g., a supervisor or a diagnoser, depending on the specific application.

Due to the local computation capability of each edge sensor, it will decide, based it own observation history, whether to transmit this observation to the central site or not. Such a decision mechanism is formalized as an *information transmission policy*

$$\Omega_i : \Sigma_{o,i}^* \Sigma_{o,i} \to \Sigma_{o,i} \cup \{\epsilon\}$$

That is, for each local observation $s\sigma \in \Sigma_{o,i}^* \Sigma_{o,i}$, Ω_i will decide whether to transmit the observation of σ, i.e., $\Omega_i(s\sigma) = \sigma$, or not, i.e., $\Omega_i(s\sigma) = \epsilon$. The above definition of information transmission policy is history-dependent. In practice, such a policy needs to be implemented in finite memory, which can be represented as a pair (a finite transducer)

$$\Omega_i = (\mathbf{A}_i, L_i), \tag{1}$$

where $\mathbf{A}_i = (X_i, \Sigma, f_i, x_{0,i})$ is a DFA, called the *sensor automaton*, such that

- $L(\mathbf{A}_i) = \Sigma^*$; and
- $\forall x \in X_i, \sigma \notin \Sigma_{o,i} : f_i(x, \sigma) = x$.

and $L_i : X_i \times \Sigma_{i,o} \to \{Y, N\}$ is a labeling function that determines whether the current observable event is transmitted or not. Here, we assume the event domain of $\mathbf{A}_i$ is Σ for the sake of simplicity, but it can only update its sensor state upon the occurrences of its locally observable events $\Sigma_{o,i}$. Also, for any $\sigma \in \Sigma_{o,i}$, $L_i(x, \sigma) = Y$ means that the occurrence of event σ will be transmitted if the edge sensor is at state x, while $L_i(x, \sigma) = N$ represents the opposite. Then for each state $x \in X_i$, we denote by

$$\theta_i(x) = \{\sigma \in \Sigma_{o,i} : L_i(x, \sigma) = Y\}$$

the set of events whose occurrences will be transmitted by Ω_i at sensor state x. Hereafter in the paper, an information transmission policy will be considered as a pair $\Omega_i = (\mathbf{A}_i, L_i)$ rather than a language-based mapping.

C. Central Observation by Collecting Edge Information

Now, let $\bar{\Omega} = [\Omega_1, \Omega_2, \ldots, \Omega_n]$ be the collection of information transmission policies for all edge sensors, where each $\Omega_i = (\mathbf{A}_i, L_i)$ sends information to the central site distributively. Therefore, from the central site's point of view, the information that receives from all edge sensors can be specified by an $\bar{\Omega}$-induced new projection $P_{\bar{\Omega}} : \Sigma^* \to \Sigma_o^*$, which is defined recursively by:

- $P_{\bar{\Omega}}(\epsilon) = \epsilon$; and
- for any $s \in \Sigma^*, \sigma \in \Sigma$, we have

$$P_{\bar{\Omega}}(s\sigma) = \begin{cases} P_{\bar{\Omega}}(s)\sigma & \text{if } \sigma \in \cup_{i \in \mathcal{I}} \theta_i(f_i(x_{0,i}, s)) \\ P_{\bar{\Omega}}(s) & \text{otherwise} \end{cases}$$

Above definition says that an event can be observed by the central site if there exists (at least) one edge sensor that can observe this event and its information transmission policy will transmit this observation.

Therefore, for any string $s \in L(\mathbf{G})$ generated by the system, we define

$$\mathcal{E}_{\bar{\Omega}}^G(s) := \{\delta(q_0, t) \in Q : \exists t \in L(\mathbf{G}) \text{ s.t. } P_{\bar{\Omega}}(s) = P_{\bar{\Omega}}(t)\} \tag{2}$$

as the central *state estimate* of the system. Clearly, for strings $s, t \in L(\mathbf{G})$, if $P_{\bar{\Omega}}(s) = P_{\bar{\Omega}}(t)$, then $\mathcal{E}_{\bar{\Omega}}^G(s) = \mathcal{E}_{\bar{\Omega}}^G(t)$.

D. Problem Formulations

In our setting, the computation of each edge sensor only aims to determine which observable event to transmit in order to save bandwidth usage. The ultimate objective is still to make sure that the central site will have sufficient information for the purpose of decision making. In this work, instead of considering specific objectives, e.g., safety or opacity, we consider a general class of decision-making requirement called the *distinguishability*.

Definition 1: (Distinguishability) Let $T \subseteq Q \times Q$ be the specification imposed on the system $\mathbf{G} = (Q, \Sigma, \delta, q_0)$, and $\bar{\Omega} = [\Omega_1, \Omega_2, \ldots, \Omega_n]$ be a set of information transition policies. We say that $\mathbf{G}$ is distinguishable w.r.t. $\bar{\Omega}$ and T if for any string $s \in L(\mathbf{G})$, we have

$$(\mathcal{E}_{\bar{\Omega}}^G(s) \times \mathcal{E}_{\bar{\Omega}}^G(s)) \cap T = \emptyset. \tag{3}$$

In the above definition, specification T is a set of state pairs for which the central site should always be able to distinguish, i.e., for any $(q, q') \in T$, if q is possible in the state estimate, then q' should not be included in it, and vice versa.

In this work, we consider two different problems for the distinguishability. One is the *verification problem*, which assumes that the information transmission policy for each edge sensor has already been designed, and we want to verify whether or not such policies fulfill the distinguishability requirement at the central site. The other one is the *synthesis problem*, namely how to design an information transmission policy for each edge sensor such that they are verified to be distinguishable. These two problems are formulated as follows.

Problem 1: (Verification Problem) Let $\mathbf{G}$ be a system equipped with a set of edge sensors associate with information transmission policies $\bar{\Omega} = [\Omega_1, \Omega_2, \ldots, \Omega_n]$ and $T \subseteq Q \times Q$ be a specification. Verify whether or not $\mathbf{G}$ is distinguishable w.r.t. $\bar{\Omega}$ and T.

Problem 2: (Synthesis Problem) Let $\mathbf{G}$ be a system equipped with a set of edge sensors with local observations $\Sigma_{o,i}, i \in \mathcal{I}$ and $T \subseteq Q \times Q$ be a specification. Find information transmission policies $\bar{\Omega} = [\Omega_1, \Omega_2, \ldots, \Omega_n]$ for edge sensors such that $\mathbf{G}$ is distinguishable w.r.t. $\bar{\Omega}$ and T, and $\bar{\Omega} = [\Omega_1, \Omega_2, \ldots, \Omega_n]$ transmit events as less as possible.

III. Verification of Information Transmission Policies

In this section, we solve the verification problem (Problem 1) for a given set of information transmission policies by two different approaches. The first approach is based on building the observer from the central site's point of view. This approach is practical when the central site wants to compute the state estimate online. However, it is costly for purpose of offline verification only. Therefore, another approach based on the verifier automaton is proposed. For the sake of simplicity, hereafter in this work, we assume $\mathcal{I} = \{1, 2\}$; our approach can be easily extended to the case of the arbitrary number of edge sensors.

A. Observer-Based Approach

Let $\bar{\Omega} = [\Omega_1, \Omega_2]$ be a set of local information transmission polices, where $\Omega_i = (\mathbf{A}_i, L_i)$ with $\mathbf{A}_i = (X_i, \Sigma, f_i, x_{0,i})$. To build the observer of $\mathbf{G}$, we first define a new automaton

$$\mathbf{V} = \mathbf{G} \times \mathbf{A}_1 \times \mathbf{A}_2 = (Q_V, \Sigma, \delta_V, q_{0,V}) \tag{4}$$

where $Q_V \subseteq Q \times X_1 \times X_2$ is the set of states, Σ is the set of events, $q_{0,V} = (q_0, x_{0,1}, x_{0,2})$ is the initial state, and $\delta_V : Q_V \times \Sigma \to Q_V$ is the transition function defined by: for any $q_V = (q, x_1, x_2) \in Q_V$, $\delta_V(q_V, \sigma) =$

$(\delta(q,\sigma), f_1(x_1,\sigma), f_2(x_2,\sigma))$. Note that, since we have assumed that $L(A_i) = \Sigma^*$ for $i \in \mathcal{I}$, $\delta_V(q_V,\sigma)!$ iff $\delta(q,\sigma)!$, and therefore, we have $L(\mathbf{V}) = L(\mathbf{G})$.

To compute the state estimate $\mathcal{E}^G_{\bar{\Omega}}(s)$, we construct the *observer* of $\mathbf{G}$ under information transmission policy $\bar{\Omega}$, which is defined as a new DFA

$$Obs_{\bar{\Omega}}(\mathbf{G}) = (Z, \Sigma_o, \xi, z_0), \tag{5}$$

where $Z \subseteq 2^{Q_V} \setminus \emptyset$ is the set of states, $\Sigma_o = \Sigma_{o,1} \cup \Sigma_{o,2}$ is the set of events, $\xi : Z \times \Sigma_o \to Z$ is the partial transition function defined by: for any $z \in Z, \sigma \in \Sigma$, we have

$$\xi(z,\sigma) = UR(NX_\sigma(z)),$$

where for any $z \in Z$, we have

$$NX_\sigma(z) = \left\{ q'_V \in Z : \begin{array}{c} \exists q_V = (q,x_1,x_2) \in z \text{ s.t.} \\ \sigma \in \theta_1(x_1) \cup \theta_2(x_2), q'_V = \delta_V(q_V,\sigma) \end{array} \right\} \tag{6}$$

and $UR(z)$ is defined inductively by:

- $z \subseteq UR(z)$; and
- for any $q_V = (q,x_1,x_2) \in z$ and $\sigma \notin \theta_1(x_1) \cup \theta_2(x_2)$, we have $\delta_V(q_V,\sigma) \in UR(z)$.

The initial state z_0 is defined by $UR(\{q_{0,V}\})$. Intuitively, $NX_\sigma(z)$ is the set of states that can be reached from some state in z immediately by transmitted event σ, and $UR(z)$ is the set of states that can be reached unobservably from some state in z. Note that the above construction of the observer is different from the standard construction of observer under natural projection; see, e.g., section 4.8 in [30]. The main difference is that whether or not each event is observable from the central site's point of view is changed dynamically depending on the states of the information transmission policies. Therefore, we cannot find a closed-form expression of $UR(z)$, and it has to be defined inductively.

For any $z \in Z \subseteq 2^{Q_V}$, we denote by $Q(z) = \{q \in Q : (q, z_1, z_2) \in z\} \in 2^Q$ the set of states in the first component of z. The following result shows that the above proposed observer construction indeed computes the desired state estimate.

Proposition 1: The observer $Obs_{\bar{\Omega}}(\mathbf{G})$ has the following properties:

(i) $P_{\bar{\Omega}}(L(\mathbf{G})) = L(Obs_{\bar{\Omega}}(\mathbf{G}))$; and
(ii) For any string $s \in L(\mathbf{G})$, we have

$$\mathcal{E}^G_{\bar{\Omega}}(s) = Q(\xi(z_0, P_{\bar{\Omega}}(s)))$$

.

The above result also leads to our first approach for the verification of distinguishability.

Theorem 1: System $\mathbf{G}$ is distinguishable w.r.t. $\bar{\Omega}$ and $T \subseteq Q \times Q$ if and only if $\forall z \in Z : (Q(z) \times Q(z)) \cap T = \emptyset$.

We illustrate the observer-based verification by the following example.

Example 1: Let us consider system $\mathbf{G}$ and information transmission policies $[\Omega_1, \Omega_2]$ as shown in Figure 2, where $\Sigma_{o,1} = \{o, a\}$ and $\Sigma_{o,2} = \{o, b, c\}$. We first compute $\mathbf{V} = \mathbf{G} \times \mathbf{A}_1 \times \mathbf{A}_2$ which is shown in Figure 3. Then the observer

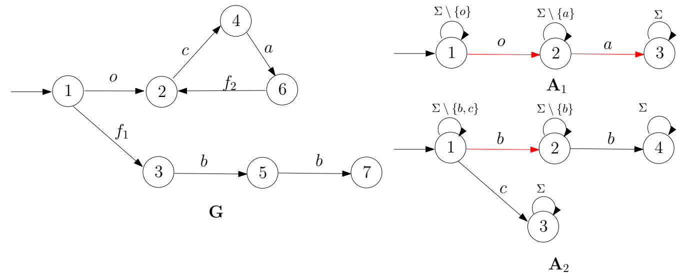

Fig. 2. System $\mathbf{G}$ and information transmission policies $\mathbf{A}_1$ and $\mathbf{A}_2$ with $L_1(1,o) = Y$, $L_1(2,a) = Y$, $L_2(1,b) = Y$, $L_2(1,c) = N$, and $L_2(2,b) = N$. The transmitted events are denoted by red line in the figures.

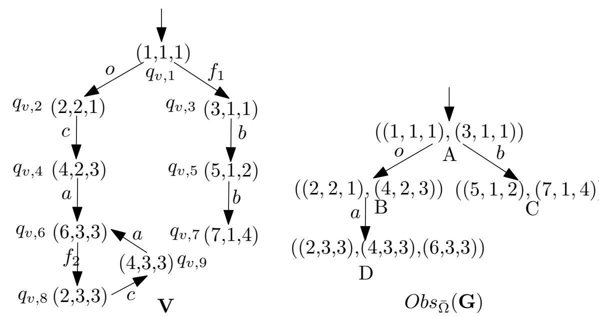

Fig. 3. Product automaton $\mathbf{V}$ and observer $Obs_{\bar{\Omega}}(\mathbf{G})$.

$Obs_{\bar{\Omega}}(\mathbf{G})$ is shown in Figure 3. Suppose $T = Q_1 \times Q_2 = \{1,2,6\} \times \{4,5,7\}$. Then $\mathbf{G}$ is not distinguishable, since at state D of $Obs_{\bar{\Omega}}(\mathbf{G})$, we have $(Q(D) \times Q(D)) \cap T = \{2,4,6\} \times \{2,4,6\} \cap T = \{(2,4),(6,4)\} \neq \emptyset$. Therefore, by Theorem 1 the given information transmission policies $[\Omega_1, \Omega_2]$ cannot ensure the distinguishablility of $\mathbf{G}$.

B. Verifier-Based Approach

By Proposition 1, since the observer generates all observable strings from the central site's point of view and for any observed string, it leads to a state whose first component is the state estimate. The observer is not only valid for offline verification, but also needed for online estimation. However, the observer is not efficient if one only considers the offline verification as it is exponential in the size of the system. Here, we further propose to use the verifier structure to check distinguishability under edge sensors.

Still, let $\mathbf{G} = (Q, \Sigma, \delta, q_0)$ be the system, $\bar{\Omega} = [\Omega_1, \Omega_2]$ be a set of local information transmission polices, where $\Omega_i = (\mathbf{A}_i, L_i)$ with $\mathbf{A}_i = (X_i, \Sigma_{o,i}, f_i, x_{0,i})$ for $i = 1, 2$. Then the verifier is a new DFA

$$\mathbf{R} = (Q_R, \Sigma_R, \delta_R, q_{0,R}), \tag{7}$$

where

- $Q_R \subseteq Q_V \times Q_V$ is the set of states;
- $\Sigma_R = (\Sigma \times \Sigma) \cup (\{\epsilon\} \times \Sigma) \cup (\Sigma \times \{\epsilon\})$ is the set of events;
- $\delta_R : Q_R \times \Sigma_R \to Q_R$ is the transition function defined by: for any $q_R = (q_V, q'_V) = (q, x_1, x_2, q', x'_1, x'_2) \in Q_R$ and $\sigma \in \Sigma$
  - If $\sigma \in (\theta_1(x_1) \cup \theta_2(x_2)) \cap (\theta_1(x'_1) \cup \theta_2(x'_2))$, then

$$\delta_R(q_R, (\sigma,\sigma)) = (\delta_V(q_V,\sigma), \delta_V(q'_V,\sigma))$$

– If $\sigma \in (\theta_1(x_1) \cup \theta_2(x_2)) \setminus (\theta_1(x_1') \cup \theta_2(x_2'))$, then

$$\delta_R(q_R, (\epsilon, \sigma)) = (q_V, \delta_V(q_V', \sigma))$$

– If $\sigma \in (\theta_1(x_1') \cup \theta_2(x_2')) \setminus (\theta_1(x_1) \cup \theta_2(x_2))$, then

$$\delta_R(q_R, (\sigma, \epsilon)) = (\delta_V(q_V, \sigma), q_V')$$

– If $\sigma \notin (\theta_1(x_1) \cup \theta_2(x_2)) \cup (\theta_1(x_1') \cup \theta_2(x_2'))$, then

$$\delta_R(q_R, (\epsilon, \sigma)) = (q_V, \delta_V(q_V', \sigma))$$

$$\delta_R(q_R, (\sigma, \epsilon)) = (\delta_V(q_V, \sigma), q_V')$$

The construction of automaton $\mathbf{R}$ is motivated by the verifier construction in the static or dynamic observation setting. Here our construction is more involved as we need to consider local information transmission policies to determine the observability of each event at the central site. Intuitively, our construction tracks a pair of strings that are observational equivalent from the central receiver's point of view, i.e., for any string $s = (s_1, s_2) \in L(\mathbf{R})$, we have $P_{\bar{\Omega}}(s_1) = P_{\bar{\Omega}}(s_2)$. Furthermore, all such pairs are included in the structure if $\mathbf{R}$, i.e., for any strings $s_1, s_2 \in L(\mathbf{G})$, if $P_{\bar{\Omega}}(s_1) = P_{\bar{\Omega}}(s_2)$, then there exists a string $s \in L(\mathbf{R})$ such that $s = (s_1, s_2)$. These two properties are rather straightforward by the construction of $\mathbf{R}$ and one can show easily by induction. Specifically, by tracking a pair of strings that are observational equivalent the two components of each state of $\mathbf{R}$ are used to estimate states in the original system. All possible confusing states are listed under the given local information transmission polices, and thus can be used to check whether the given policies ensure distinguishability. Our main result is given in the following.

Theorem 2: System $\mathbf{G}$ is distinguishable w.r.t. $\bar{\Omega}$ and $T \subseteq Q \times Q$ if and only if

$$\forall\ q_R = (q, x_1, x_2, q', x_1', x_2') \in Q_R : (q, q') \notin T. \quad (8)$$

We illustrate the verification of distinguishability using the verifier by the following example.

Example 2: Let us again consider system $\mathbf{G}$ and information transmission policies $\mathbf{A}_1$ and $\mathbf{A}_2$ shown in Figure 2 with $\Sigma_{o,1} = \{o, a\}$ and $\Sigma_{o,1} = \{o, b, c\}$. We construct the verifier automaton $\mathbf{R}$ as shown in Figure 4. Suppose again $T = Q_1 \times Q_2 = \{1, 2, 6\} \times \{4, 5, 7\}$. Here, we employ Theorem 2 to verify whether or not the system is distinguishable w.r.t. $\bar{\Omega}$ and T. As shown in Figure 4, for state $(q_{v,2}, q_{v,4}) = ((2, 2, 1), (4, 2, 3)) \in Z$, we have $(2, 4) \in T$. We thus get that $\mathbf{R}$ is not distinguishable w.r.t. $\bar{\Omega}^*$ and T. Therefore, by Theorem 2, $\mathbf{G}$ is not distinguishable w.r.t. $\bar{\Omega}$ and T.

IV. Synthesis of Optimal Information Transmission Policies

In the previous section, we have investigated how to verify whether a given set of information transmission policies ensures distinguishability or not. In this section, we further investigate how to *synthesize* a set of information transmission policies for edge sensors such that distinguishability is fulfilled by construction.

As we discussed early, each information transmission policy is language-based in general; therefore, the solution

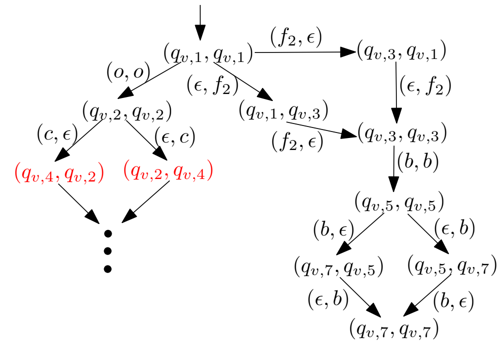

Fig. 4. The verifier automaton $\mathbf{R}$.

space for searching optimal policies may be unbounded. Further, each edge sensor essentially transmits information collaboratively in the sense that one edge sensor should take other sensors' strategies into account to synthesize its strategy. However, this local strategy will again affect the optimality of other strategies. This information dependency may require infinite iterations, which again, results in unbounded solution space. In order to resolve the above issues, here we propose to restricting our attention to *observer-based strategies*, i.e., the decision space for each edge sensor is restricted to its local observer.

Specifically, for each edge sensor with observable events $\Sigma_{o,i}$, its local observer is a new DFA

$$Obs_i(\mathbf{G}) = (Z_i, \Sigma_{o,i}, \xi_i, z_{0,i}), \quad (9)$$

where $Z_i \subseteq 2^Q$ is the set of states, $\xi_i : Z_i \times \Sigma_{o,i} \to Z_i$ is the transition function defined by: for any $z \in Z_i$ and $\sigma \in \Sigma_{o,i}$, we have

$$\xi_i(z, \sigma) = \{q' \in Q : \exists q \in z, w \in (\Sigma \setminus \Sigma_{o,i})^* \text{ s.t. } q' = \delta(q, \sigma w)\}$$

and $z_{0,i} = \{q' \in Q : \exists w \in (\Sigma \setminus \Sigma_{o,i})^* \text{ s.t. } q' = \delta(q_0, w)\}$ is the initial state.

Note that $Obs_i(\mathbf{G})$ is different from the central observer $Obs(\mathbf{G})$ used for the purpose of online estimate and verification. In particular, $Obs(\mathbf{G})$ only can be constructed when each local transmission policy is specified. However, here $Obs_i(\mathbf{G})$ is built based on its local sensing capability $\Sigma_{o,i}$ without the need of specifying its transmission policy. Our purpose is to consider $Obs_i(\mathbf{G})$ as the state-space for specifying the information transmission policy of edge sensor $i \in \mathcal{I}$. We denote by $\widetilde{Obs_i}(\mathbf{G})$ the DFA obtained by adding self-loops for missing events at each state in $Obs_i(\mathbf{G})$ such that $L(\widetilde{Obs_i}(\mathbf{G})) = \Sigma^*$. Then for each edge sensor $i \in \mathcal{I}$, $\Omega_i = (\widetilde{Obs_i}(\mathbf{G}), L_i)$ is called an *observer-based* information transmission policy. Hereafter, we assume that $\mathbf{G}$ is distinguishable if each L_i always chooses to transmit observed events. Otherwise, the synthesis problem will trivially have no solution.

Now we are ready to provide the algorithm for synthesizing information transmission policies, which is given in Algorithm 1. The basic idea is to mark as many of the observable events of the local sensors not transmissible as

Algorithm 1: EDGE-SENSOR-TRANS-POLICY

input : $\mathbf{G}$, $\Sigma_{o,1}, \Sigma_{o,2}$ and T
output: $\bar{\Omega} = [\Omega_1, \Omega_2] = [(\mathbf{A}_1, L_1), (\mathbf{A}_2, L_2)]$

1 **for** $i \in \{1,2\}$ **do**
2     $\mathbf{A}_i \leftarrow \widetilde{Obs}_i(\mathbf{G}) = (X_i, \Sigma, \tilde{f}_i, x_{0,i})$
3   for any $x \in X_i, \sigma \in \Sigma$, set $L_i(x,\sigma) = \begin{cases} Y & \text{if } \sigma \in \Sigma_{o,i} \wedge f_i(x,\sigma)! \\ N & \text{otherwise} \end{cases}$
4 **for** $W \subseteq Tr_Y(\mathbf{A}_1) \cup Tr_Y(\mathbf{A}_2)$ **do**
5   for any $(x,\sigma) \in W$ such that $f_i(x,\sigma)!$, set $L_i(x,\sigma) \leftarrow N$
6   Verify if $\mathbf{G}$ is distinguishable w.r.t. $[((\mathbf{A}_1, L_1), (\mathbf{A}_2, L_2))]$ and T
7   **if G** *is not distinguishable* **then**
8   for any $(x,\sigma) \in W$ such that $f_i(x,\sigma)!$, set $L_i(x,\sigma) \leftarrow Y$
      else
9   **break** the for-loop and **go to** line 4
10 **return** $\bar{\Omega} \leftarrow [(\mathbf{A}_1, L_1), (\mathbf{A}_2, L_2)]$

possible while ensuring the distinguishability of the system. For each $\Omega_i = (\mathbf{A}_i, L_i)$, we define

$$Tr_Y(\mathbf{A}_i) = \{(x,\sigma) \in X_i \times \Sigma_{o,i} : L_i(x,\sigma) = Y\}$$

as the set of transitions corresponding to event transmissions in $\mathbf{A}_i$. Then Algorithm 1 works as follows. Initially, we construct the sensor automaton $\mathbf{A}_i$ for each $i \in \mathcal{I}$ based on the local observer of $\mathbf{G}$ (line 2) and allow them to transmit all observable events (lines 3). Then for any *subset* of transitions W associated with event transmissions, we test whether or not by making the corresponding observable events non-transmitted (line 5), $\mathbf{G}$ is still distinguishable w.r.t. $[((\mathbf{A}_1, L_1), (\mathbf{A}_2, L_2))]$ and T (line 6). If so, we will keep W non-transmitted, break the current for-loop, and repeat searching for subsets of transitions. Therefore, the only case we reach the final solution in line 10 is that, for any subset W, we cannot make the transitions non-transmitted, which means the current transmission policies cannot be improved anymore.

For any two observer-based information transmission policies $\Omega_i = (\mathbf{A}_i, L_i)$ and $\Omega'_i = (\mathbf{A}'_i, L'_i)$ for $i \in \mathcal{I}$, we write $\Omega'_i \subseteq \Omega_i$ if for any $s\sigma \in P_i(L(\mathbf{G}))$, we have

$$L'_i(f'_i(x_{0,i}, s), \sigma) = Y \Rightarrow L_i(f(x_{0,i}, s), \sigma) = Y$$

and write that $\Omega'_i \subset \Omega_i$ if $\Omega'_i \subseteq \Omega_i$ and there exists $s\sigma \in P_i(L(\mathbf{G}))$ such that

$$L'_i(f'(x_{0,i}, s), \sigma) = N \wedge L_i(f(x_{0,i}, s), \sigma) = Y$$

Then, for any $\bar{\Omega} = [\Omega_1, \Omega_2]$ and $\bar{\Omega}' = [\Omega'_1, \Omega'_2]$, we denote by $\bar{\Omega}' \subseteq \bar{\Omega}$ if $\forall i \in [1,n] : \Omega'_i \subseteq \Omega_i$ and by $\bar{\Omega}' \subset \bar{\Omega}$ if $\bar{\Omega}' \subseteq \bar{\Omega}$ and $\exists i \in [1,n] : \Omega'_i \subset \Omega_i]$. Then we say an observer-based information transmission policy $\bar{\Omega}$ is observer-based optimal if there does not exist another information transmission policy $\bar{\Omega}'$ s.t. $\bar{\Omega}' \subset \bar{\Omega}$.

Theorem 3: The observer-based information transmission policy $\bar{\Omega} = [\Omega_1, \Omega_2]$ obtained by Algorithm 1 is distinguishable and observer-based optimal, i.e., it solves Problem 2.

Remark 1: Note that in Algorithm 1, we test for all possible *subsets* of transitions W whether or not the system is distinguishable by setting their communication labels are set from Y to N. Such an approach requires to enumerate, in the worst-case, the power-set of all transitions to determine whether or not the current solution is optimal. One may ask why we do not take a greedy search by test transitions one-by-one. This is because it is known that there is no so-called monotonicity property in communications. It is possible that the system is not distinguishable by removing each of the communications individually but becomes distinguishable by removing their combinations. Therefore, a greedy search may not yield an optimal solution.

Example 3: Let us again consider system $\mathbf{G}$ in Figure 2 with $\Sigma_{o,1} = \{o,a\}$ and $\Sigma_{o,1} = \{o,b,c\}$, and specification $T = Q_1 \times Q_2 = \{1,3,5\} \times \{6,7\}$. Here, we employ Algorithm 1 to synthesize a set of information transmission polices $\bar{\Omega}^* = \{\Omega_1, \Omega_2\}$ such that the system is distinguishable w.r.t. $\bar{\Omega}^*$ and T.

Step 1: Construct observers $\mathbf{A}_1 = \widetilde{Obs}_1(\mathbf{G}) = (X_1, \Sigma_{o,1}, f_1, x_{0,1})$ with $X_1 = \{x_1^1, x_1^2\}$ and $\mathbf{A}_2 = \widetilde{Obs}_2(\mathbf{G}) = (X_2, \Sigma_{o,2}, f_2, x_{0,2})$ with $X_2 = \{x_2^1, x_2^2, x_2^3, x_2^4\}$ by (9). For any $x \in X_i$ and $\sigma \in \Sigma_{o,i}$, let $L_i(x,\sigma) = Y$ if $f_i(x,\sigma)!$. Hence $Tr_Y(\mathbf{A}_1) = \{(x_1^1, o), (x_1^2, a)\}$ and $Tr_Y(\mathbf{A}_2) = \{(x_2^1, o), (x_2^1, b), (x_2^2, c), (x_2^3, b)\}$.

Step 2:

1) Let $W = \{(x_2^1, o)\}$ and $L_1(x_2^1, o) = N$, and $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$. Construct $\mathbf{R}$ w.r.t $\bar{\Omega} = [\Omega_1, \Omega_2]$ by (7). $\mathbf{R}$ satisfies (8), so we keep $L_2(x_2^1, o) = N$ and let $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$.

2) Let $W = \{(x_1^1, o)\}$ and $L_1(x_1^1, o) = N$, and $\Omega_1 \leftarrow (\mathbf{A}_1, L_1)$. Construct $\mathbf{R}$ w.r.t $\bar{\Omega} = [\Omega_1, \Omega_2]$ by (7). $\mathbf{R}$ dissatisfies (8) since $\exists q_r \in Q_R$. s.t. $Q(q_r) \cap T = \{1\} \times \{6\} \neq \emptyset$. Hence we let $L_1(x_1^1, o) = Y$ and $\Omega_1 \leftarrow (\mathbf{A}_1, L_1)$.

3) Let $W = \{(x_1^2, a)\}$ and $L_1(x_1^2, a) = N$, and $\Omega_1 \leftarrow (\mathbf{A}_1, L_1)$. Construct $\mathbf{R}$ w.r.t $\bar{\Omega} = [\Omega_1, \Omega_2]$ by (7). $\mathbf{R}$ satisfies (8), so we keep $L_1(x_1^2, a) = N$ and let $\Omega_1 \leftarrow (\mathbf{A}_1, L_1)$.

4) Let $W = \{(x_2^1, b)\}$, $L_2(x_2^1, b) = N$ and $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$. Construct $\mathbf{R}$ w.r.t $\bar{\Omega} = [\Omega_1, \Omega_2]$ by (7). $\mathbf{R}$ satisfies (8), so we keep $L_2(x_2^1, b) = N$ and let $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$.

5) Let $W = \{(x_2^2, c)\}$, $L_2(x_2^2, c) = N$ and $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$. Construct $\mathbf{R}$ w.r.t $\bar{\Omega} = [\Omega_1, \Omega_2]$ by (7). $\mathbf{R}$ satisfies (8), so we keep $L_2(x_2^2, c) = N$ and let $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$.

6) Let $W = \{(x_2^3, b)\}$, $L_2(x_2^3, b) = N$ and $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$. Construct $\mathbf{R}$ w.r.t $\bar{\Omega} = [\Omega_1, \Omega_2]$ by (7). $\mathbf{R}$ dissatisfies (8) since $\exists q_r \in Q_R$. s.t. $Q(q_r) \cap T = \{5\} \times \{7\} \neq \emptyset$. Hence we let $L_2(x_2^3, b) = Y$ and $\Omega_2 \leftarrow (\mathbf{A}_2, L_2)$.

7) Similarly, we will check the cases $W = \{(x_1^1, o), (x_1^2, a)\}$, $W = \{(x_1^1, o), (x_2^1, o)\}, \ldots$, $W = Tr_Y(\mathbf{A}_1) \cup Tr_Y(\mathbf{A}_2) = \{(x_1^1, o), (x_1^2, a), (x_2^1, o), (x_2^1, b), (x_2^2, c)\}$.

Step 3: Output: $\bar{\Omega} \leftarrow [\Omega_1, \Omega_2]$. $\mathbf{A}_1$ and $\mathbf{A}_2$ are shown in Fig. 5 with labels $L_1(x_1^1, o) = Y$, $L_1(x_1^2, a) = N$, $L_2(x_2^1, o) = N$, $L_2(x_2^1, b) = N$, $L_2(x_2^2, c) = N$, and $L_2(x_2^3, b) = Y$.

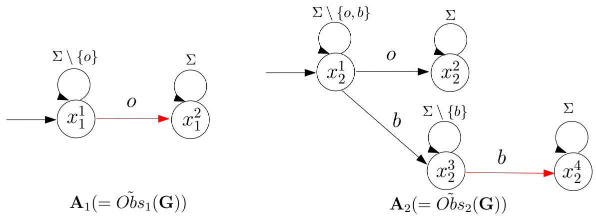

Fig. 5. Sensor automata $\mathbf{A}_1$ and $\mathbf{A}_2$ with $L_1(x_1^1, o) = Y$, $L_1(x_1^2, a) = N$, $L_2(x_2^1, o) = N$, $L_2(x_2^1, b) = N$, $L_2(x_2^2, c) = N$, and $L_2(x_2^3, b) = Y$.

V. Conclusion

We proposed a new architecture to tackle the problem of distributed sensing and information transmission for the purpose of centralized decision making in partially observed discrete-event systems with edge sensors. Information transmission policies for the edge sensors are given to decide which observed event should to be transmitted to the central site. Then, we also proposed two approaches to verify whether the system is centralized distinguishable under a given set of sensor transmission policies. One approach is based on building an observer from the central site. Another one is based on a verifier structure. Finally, we provided an algorithm to synthesize a set of optimal information transmission policies for edge sensors such that distinguishability is fulfilled by construction.

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