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Enhancing Event-Separation Properties for Event-Triggered Consensus with Disturbances

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Abstract— **In the event-triggered control of multi-agent systems (MASs), external disturbances are prevalent in practice and may lead to excessive triggering events (often referred to as Zeno behavior), which poses problems for practical implementation. Therefore, it is essential to enhance event-separation properties when the MAS is subjected to disturbances. In this context, this article is concerned with the event-triggered consensus of MASs in the presence of external disturbances. Distributed dynamic event-triggered (DET) control strategies are proposed based on the sampled information of neighboring agents. It has been theoretically demonstrated that, under the designed DET sampling strategies, the MAS can achieve bounded consensus and strictly positive minimum inter-event times are guaranteed. The effectiveness of the theoretical results is validated by numerical results.**

I. Introduction

Over the past few decades, the consensus problem of multi-agent systems (MASs) has attracted significant attention from numerous researchers due to its broad applications, including aircraft formation control, sensor networks, and electrical power grids [1]. Through the communication between agents, consensus control focuses on designing distributed controllers that enable agents within a network to achieve a desired agreement in some sense. In practical control systems, resource constraints widely exist such as limited communication bandwidth and computational capacity. Therefore, it is essential to reduce the communication burdens in the MAS consensus control. To address this challenge, event-triggered control schemes have been extensively studied for their ability to reduce the frequency of information exchange and controller updates [2]–[5]. For instance, event-triggered protocols were designed in [3] to achieve consensus based on locally sampled state information. Consensus of linear MASs on directed graphs was systematically studied in [4] and innovative adaptive event-triggered state-feedback protocols were presented.

In MAS event-triggered control, one of the most concerned topics is to design effective sampling mechanisms that determine asynchronous triggering sequences for agents. Consequently, the properties of these triggering sequences have received considerable attention, particularly in the following two primary aspects. Firstly, reducing the number of triggered events is essential for saving communication resources and thus it is crucial to explore strategies that can increase the average inter-event times. Secondly, due to practical limitations on the frequency of controller updates, the minimum inter-event times (MIETs) should be strictly positive. Regarding the first aspect, dynamic event-triggered (DET) mechanisms have demonstrated their effectiveness in extending the average inter-event times [6]–[10]. In [9], it had been shown that compared with static cases, the frequency of triggers can be greatly reduced. In the framework of fully distributed control, DET mechanisms were designed in [10] to address the consensus problem for both leaderless and leader-follower cases, respectively. For the second aspect, previous studies have often focused on eliminating Zeno behavior. However, even if Zeno behavior is excluded, there may still exist two consecutive event times that are arbitrarily close to one another in an infinite time horizon, which implies that the algorithm may still be unimplementable. To address this limitation, many recent works have aimed to establish a strictly positive lower bound of MIETs [11]–[16] and the properties related were called *event-separation properties* in [17]. In [11], [12], event-triggering conditions were proposed that only require periodic evaluation, thereby ensuring strictly positive inter-event times. Additionally, works such as [13], [14] designed triggering functions by introducing a clock variable, and the lower bound of MIETs is given by the time taken for the variable to evolve from a positive constant to zero. In [15], [16], the lower bound of inter-event times was ensured by an enforced time, before which the event-triggering conditions were not checked.

In this paper, to save communication resources, we employ DET mechanisms to solve the consensus problem of MASs. In some previous works on DET mechanisms, such as [7]–[10], it was merely proved that the Zeno behavior can be excluded, while no uniform positive MIETs can be guaranteed. On the contrary, we rigorously prove in this paper that the DET mechanisms can inherently ensure positive MIETs and thus the event-separation properties of DET mechanisms are established. Moreover, it should be noted that while the previous results demonstrated that Zeno behavior can be excluded or positive MIETs can be guaranteed, most of these studies did not consider the impact of external disturbances. As highlighted in [17], even arbitrarily small disturbances can lead to an arbitrarily small even interval, making an event-triggered strategy impractical. Therefore, addressing the influence of external disturbances is crucial to ensure

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the robustness and feasibility of event-triggered mechanisms in MASs. Recently, the results in [18]–[20] considered the influence of disturbances for event-triggered control of MAS. Nevertheless, the event-triggered mechanisms in [18] were centralized and only Zeno behavior was excluded in [18]–[20]. In this work, on the basis of the DET mechanisms, some extra parameters are introduced to ensure positive MIETs in the presence of external disturbances. It can be demonstrated that these introduced parameters can help ensure positive MIETs and enhance the event-separation properties of DET mechanisms, which is the core innovation of this work.

Notation: Let $\mathbb{N}$ and $\mathbb{R}$ denote the sets of nonnegative integers and real numbers, respectively. An $n \times n$ identity matrix is expressed as $\mathbf{I}_n$, whereas a column vector in n-dimensional space containing exclusively unit values is denoted by $\mathbf{1}_n$ (the subscript is omitted if no confusion).

Graph Theory: Graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ is employed to describe the communication topology between agents. The node set $\mathcal{V} = \{1, 2, \ldots, N\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent N agents and the communication links between agents, respectively. Specifically, if $(j, i) \in \mathcal{E}$, it indicates that agent i can send information to agent j, and then agent j is said to be a neighboring agent of agent i. Set $\mathcal{N}_i$ denotes all neighbors of agent i. For the graph $\mathcal{G}$, the adjacency matrix is denoted as $\mathcal{A} \triangleq [a_{ij}]_{N\times N}$ with $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. Moreover, the Laplacian matrix is denoted by $\mathcal{L} \triangleq [l_{ij}]_{N\times N}$, where $l_{ii} = \sum_{k\in\mathcal{N}_i} a_{ik}$ for $i \in \mathcal{V}$, and $l_{ij} = -a_{ij}$ for $i, j \in \mathcal{V}$ with $i \neq j$. If there holds $a_{ij} = a_{ji}$ for all $(i, j) \in \mathcal{E}$, then we say that $\mathcal{G}$ is undirected. If there always exists a path for any two distinct nodes in an undirected graph, then we say that $\mathcal{G}$ is connected.

Assumption 1. For the MASs considered in this paper, the associated graph $\mathcal{G}$ is undirected and connected.

Lemma 1 ([21]). *Assume that the graph $\mathcal{G}$ to be undirected. Denote the N eigenvalues of the Laplacian $\mathcal{L}$ as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$. If $\mathcal{G}$ is also connected, then $\lambda_1 = 0$ is the simple zero eigenvalue and $\lambda_2 = \min_{\mathbf{1}^{\mathrm{T}}x=0, x\neq 0} \frac{x^{\mathrm{T}}\mathcal{L}x}{x^{\mathrm{T}}x} > 0$ is the smallest nonzero eigenvalue.*

II. Problem Formulation

In this paper, we consider an MAS affected by external disturbances and the dynamics of the N agents are given by

$$\dot{x}_i(t) = u_i(t) + v_i(t),\ \tilde{x}_i(t) = x_i(t) + w_i(t),\ i \in \mathcal{V}, \quad (1)$$

where $x_i(t) \in \mathbb{R}$, $u_i(t) \in \mathbb{R}$ are the state and input signal of agent i, respectively; $v_i(t) \in \mathbb{R}$ is the process disturbance and $w_i(t) \in \mathbb{R}$ is the measurement noise. Without loss of generality, here we consider the case that each agent has a single state, while it is straightforward to generalize the developments in this section to the case of multiple states (i.e., the state x_i is a vector). It should be pointed out that the states of agents are affected by measurement noises and thus only $\tilde{x}_i(t)$ is available to agent i. The following assumption provides the constraints of the external disturbances.

Assumption 2. For the disturbances in (1), there exist some positive unknown constants $\bar{v}_0^i$, $\bar{w}_1^i$ and $\bar{w}_2^i$ such that $|v_i(t)| \leq \bar{v}_0^i$, $|w_i(t)| \leq \bar{w}_1^i$ and $|\dot{w}_i(t)| \leq \bar{w}_2^i$ for all $t \geq 0$.

To more clearly highlight our focus in this paper, some definitions are firstly given as follows:

Definition 1. The consensus of MASs (1) is achieved if for any initial conditions $x_i(0)$, $i \in \mathcal{V}$, the agents' states can reach consensus asymptotically, that is,

$$\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0,\ i,\ j \in \mathcal{V}.$$

In particular, for some positive constant c_0, if there holds

$$\lim_{t\to\infty}\|x_i(t) - x_j(t)\| < c_0,\ i,\ j \in \mathcal{V},$$

then it is said that the MAS (1) reaches bounded consensus.

Definition 2. For a discrete time sequence $\{t_k\}_{k\in\mathbb{N}}$, if there exists some positive constants τ such that

$$t_{k+1} - t_k \geq \tau > 0,\ \forall k \in \mathbb{N},$$

then it is said that the time sequence $\{t_k\}$ has a strictly positive MIET.

In this paper, we attempt to design distributed event-triggered protocols and DET sampling mechanisms to generate the control input $u_i(t)$ such that the MAS (1) reaches bounded consensus under external disturbances. Furthermore, it is expected that only sampled information about agents is needed between neighboring agents so as to save communication resources and a strictly positive MIET between any two consecutive transmissions is guaranteed.

III. Main Results

A. Distributed DET mechanisms

On one hand, event-triggered communication are considered in this paper, which means that each agent only broadcasts its information at discrete times. On the other hand, affected by the measurement noise $w_i(t)$, only $\tilde{x}_i(t)$ is available to agent i. Hence, agent i can only access the sampled version of $\tilde{x}_j$, $j \in \mathcal{N}_i$. More formally, letting $\{t_k^i\}_{k\in\mathbb{N}}$ be the sequence of times at which agent i broadcasts its state to its neighbors, and the control law implemented to agent i is given by

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i(t) - \hat{\tilde{x}}_j(t)), \quad (2)$$

with $\hat{\tilde{x}}_i(t) = \tilde{x}_i(t_k^i)$, $t \in [t_k^i,\ t_{k+1}^i)$, $i \in \mathcal{V}$. Define $\tilde{e}_i \triangleq \hat{\tilde{x}}_i - \tilde{x}_i$, and dynamic triggering functions are designed as

$$f_i(t) = 4l_{ii}\tilde{e}_i^2 - \rho_i \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - o_{\mathrm{f}}^i - \omega_i\sigma_i(t), \quad (3)$$

with the dynamic variable $\sigma_i(t)$ evolving according to

$$\dot{\sigma}_i = -\delta_i\sigma_i + \rho_i \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - 4l_{ii}\tilde{e}_i^2 + o_\sigma^i, \quad (4)$$

In (3) and (4), ω_i, δ_i, o_σ^i and $\sigma_i(0)$ are any positive constants; o_{f}^i is any nonnegative constant satisfying $o_{\mathrm{f}}^i \leq o_\sigma^i$; ρ_i is any constant belonging to $(0,1)$. Let $T_k^i \triangleq \min\{t - t_k^i > 0 \mid f_i(t) \geq 0\}$, and then the sampling sequence $\{t_k^i\}$ is generated by

$$t_{k+1}^i = t_k^i + T_k^i, \ i \in \mathcal{V}, \ k \in \mathbb{N} \tag{5}$$

with $t_0^i = 0$. One of the main objectives of this paper is to prove that $T_k^i \geq \tau$, $i \in \mathcal{V}$, $\forall k \in \mathbb{N}$, for some positive constants τ, which will be presented in Theorem 2.

Proposition 1. *Under the DET mechanisms in (3)–(5), for the dynamic variable $\sigma_i(t)$, it holds that*

$$\sigma_i(t) \geq \min\left\{\sigma_i(0), \ \frac{o_\sigma^i - o_{\rm f}^i}{\delta_i + \omega_i}\right\}, \ \forall t \geq 0.$$

Proof. Since the designed event-triggering condition ensures $f_i(t) \leq 0$, we have

$$\begin{aligned}\dot{\sigma}_i &= -\delta_i\sigma_i + \rho_i \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - 4l_{ii}\tilde{e}_i^2 + o_\sigma^i \\ &\geq -(\delta_i + \omega_i)\sigma_i + o_\sigma^i - o_{\rm f}^i.\end{aligned}$$

Consider a variable satisfying

$$\dot{\beta}_i = -(\delta_i + \omega_i)\beta_i + o_\sigma^i - o_{\rm f}^i, \ \beta_i(0) = \sigma_i(0) > 0,$$

and we can obtain that

$$\beta_i(t) = \beta_i(0)\mathrm{e}^{-(\delta_i+\omega_i)t} + \frac{o_\sigma^i - o_{\rm f}^i}{\delta_i + \omega_i}\left(1 - \mathrm{e}^{-(\delta_i+\omega_i)t}\right) > 0.$$

Hence, it can be verified that $\beta_i(t) \geq \min\left\{\beta_i(0), \ \frac{o_\sigma^i - o_{\rm f}^i}{\delta_i+\omega_i}\right\}$. By the Comparison Principle [22, Lemma 3.4], it can be obtained that $\sigma_i(t) \geq \beta_i(t) > 0$ and it always holds that $\sigma_i(t) \geq \beta_i(t) \geq \min\left\{\sigma_i(0), \ \frac{o_\sigma^i - o_{\rm f}^i}{\delta_i+\omega_i}\right\}$ for all $t \geq 0$. □

B. Consensus Analysis

For later use, we first define some stacked vectors as $x \triangleq \mathrm{col}\{x_i, i \in \mathcal{V}\}$, $\hat{x} \triangleq \mathrm{col}\{\hat{x}_i, i \in \mathcal{V}\}$, $\tilde{x} \triangleq \mathrm{col}\{\tilde{x}_i, i \in \mathcal{V}\}$, $\hat{\tilde{x}} \triangleq \mathrm{col}\{\hat{\tilde{x}}_i, i \in \mathcal{V}\}$, $v \triangleq \mathrm{col}\{v_i, i \in \mathcal{V}\}$, $w \triangleq \mathrm{col}\{w_i, i \in \mathcal{V}\}$ and $\tilde{e} \triangleq \mathrm{col}\{\tilde{e}_i, i \in \mathcal{V}\}$. From (1) and (2), we have

$$\dot{\tilde{e}}_i = \dot{\hat{\tilde{x}}}_i - \dot{\tilde{x}}_i = \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j) - v_i - \dot{w}_i. \tag{6}$$

Let $\hat{\mathcal{L}} \triangleq \mathbf{I} - \frac{1}{N}\mathbf{1}\mathbf{1}^{\mathrm{T}}$ and introduce a transformation of x as $\chi \triangleq \mathrm{col}\{\chi_i, i \in \mathcal{V}\} \triangleq \hat{\mathcal{L}}x$. It follows from $\hat{\mathcal{L}}\mathcal{L} = \mathcal{L}$ that

$$\dot{\chi} = -\mathcal{L}\hat{\tilde{x}} + \hat{\mathcal{L}}v. \tag{7}$$

It can be observed that $\chi_i(t)$ represents the difference between the state $x_i(t)$ and the real-time state average $\bar{x}(t) \triangleq \frac{1}{N}\sum_{i\in\mathcal{V}} x_i(t)$ of all agents. Hence, if $\chi(t)$ is demonstrated to be bounded, then the considered MAS can achieve bounded consensus. We can derive the following theorem regarding the consensus properties of the closed-loop MAS.

Theorem 1. *Consider the MAS (1) under Assumptions 1 and 2, the protocol (2) and the DET sampling mechanisms in (3)–(5). Then the closed-loop MAS reaches bounded consensus and the dynamic variables $\sigma_i(t)$ are uniformly bounded.*

Proof. Consider a candidate Lyapunov function as

$$V = \chi^{\mathrm{T}}\chi + \sum_{i\in\mathcal{V}} \frac{\sigma_i}{2\rho_i}.$$

Take the time derivative of V along the solution of χ in (7) and we can obtain that

$$\begin{aligned}\dot{V} &= 2\chi^{\mathrm{T}}\dot{\chi} + \sum_{i\in\mathcal{V}} \frac{\dot{\sigma}_i}{2\rho_i} \\ &= -2\chi^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} + 2\chi^{\mathrm{T}}\hat{\mathcal{L}}v + \sum_{i\in\mathcal{V}} \frac{\dot{\sigma}_i}{2\rho_i} \\ &\leq -2\chi^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} + k_1\chi^{\mathrm{T}}\chi + \frac{1}{k_1}v^{\mathrm{T}}v + \sum_{i\in\mathcal{V}} \frac{\dot{\sigma}_i}{2\rho_i},\end{aligned} \tag{8}$$

where the last inequality is obtained by Young's inequality for any $k_1 \in (0, \lambda_2)$. Due to the fact that $\chi^{\mathrm{T}}\mathcal{L} = x^{\mathrm{T}}\mathcal{L}$, $x = \tilde{x} - w$, $\tilde{e} = \hat{\tilde{x}} - \tilde{x}$ and $\hat{\tilde{x}} - x = \hat{\tilde{x}} - \tilde{x} + \tilde{x} - x = \tilde{e} + w$, we have

$$\begin{aligned}&- 2\chi^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} \\ &= -2x^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} \\ &= -2\tilde{x}^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} + 2w^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} \\ &= \tilde{e}^{\mathrm{T}}\mathcal{L}\tilde{e} - \hat{\tilde{x}}^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} - \tilde{x}^{\mathrm{T}}\mathcal{L}\tilde{x} + 2w^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} \\ &= \tilde{e}^{\mathrm{T}}\mathcal{L}\tilde{e} - \hat{\tilde{x}}^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} - x^{\mathrm{T}}\mathcal{L}x - w^{\mathrm{T}}\mathcal{L}w - 2w^{\mathrm{T}}\mathcal{L}x + 2w^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} \\ &= \tilde{e}^{\mathrm{T}}\mathcal{L}\tilde{e} - \hat{\tilde{x}}^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} - \chi^{\mathrm{T}}\mathcal{L}\chi + w^{\mathrm{T}}\mathcal{L}w + 2w^{\mathrm{T}}\mathcal{L}\tilde{e}.\end{aligned} \tag{9}$$

By applying Young's inequality, it is not difficult to show

$$\begin{aligned}&2w^{\mathrm{T}}\mathcal{L}\tilde{e} \\ &= 2\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}\tilde{e}_i(w_i - w_j) \\ &\leq \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}\left[\frac{2(1-\rho_i)}{\rho_i}\tilde{e}_i^2 + \frac{\rho_i}{2(1-\rho_i)}(w_i - w_j)^2\right] \\ &= \sum_{i\in\mathcal{V}}\left(\frac{1}{\rho_i} - 1\right)2l_{ii}\tilde{e}_i^2 + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\frac{a_{ij}\rho_i}{2(1-\rho_i)}(w_i - w_j)^2 \\ &\leq \sum_{i\in\mathcal{V}}\left(\frac{1}{\rho_i} - 1\right)2l_{ii}\tilde{e}_i^2 + \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\frac{a_{ij}\rho_{\max}}{2(1-\rho_{\max})}(w_i - w_j)^2 \\ &= \sum_{i\in\mathcal{V}}\left(\frac{1}{\rho_i} - 1\right)2l_{ii}\tilde{e}_i^2 + \frac{\rho_{\max}}{1-\rho_{\max}}w^{\mathrm{T}}\mathcal{L}w,\end{aligned} \tag{10}$$

where $\rho_{\max}$ is defined as $\rho_{\max} \triangleq \max\{\rho_i, i \in \mathcal{V}\}$. Moreover, it follows from $a_{ij} = a_{ji}$ that

$$\begin{aligned}\tilde{e}^{\mathrm{T}}\mathcal{L}\tilde{e} &= \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\frac{1}{2}a_{ij}(\tilde{e}_i - \tilde{e}_j)^2 \\ &\leq \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} a_{ij}(\tilde{e}_i^2 + \tilde{e}_j^2) \\ &= \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} 2a_{ij}\tilde{e}_i^2 = \sum_{i\in\mathcal{V}} 2l_{ii}\tilde{e}_i^2,\end{aligned} \tag{11}$$

and

$$\hat{\tilde{x}}^{\mathrm{T}}\mathcal{L}\hat{\tilde{x}} = \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\frac{1}{2}a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2. \tag{12}$$

Substituting (9)–(12) into (8), we can obtain that

$$\begin{aligned}\dot{V} &\leq -\chi^{\mathrm{T}}\mathcal{L}\chi + k_1\chi^{\mathrm{T}}\chi + \frac{1}{k_1}v^{\mathrm{T}}v + \frac{1}{1-\rho_{\max}}w^{\mathrm{T}}\mathcal{L}w \\ &\quad + \sum_{i\in\mathcal{V}}\frac{1}{2\rho_i}\left[4l_{ii}\tilde{e}_i^2 - \rho_i\sum_{j\in\mathcal{N}_i}a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 + \dot{\sigma}_i\right] \\ &\leq -(\lambda_2 - k_1)\chi^{\mathrm{T}}\chi - \sum_{i\in\mathcal{V}}\frac{\delta_i\sigma_i}{2\rho_i} \\ &\quad + \sum_{i\in\mathcal{V}}\frac{o_\sigma^i}{2\rho_i} + \frac{1}{k_1}v^{\mathrm{T}}v + \frac{1}{1-\rho_{\max}}w^{\mathrm{T}}\mathcal{L}w \\ &\leq -v_0V + \sum_{i\in\mathcal{V}}\frac{o_\sigma^i}{2\rho_i} + \frac{1}{k_1}v^{\mathrm{T}}v + \frac{1}{1-\rho_{\max}}w^{\mathrm{T}}\mathcal{L}w \quad (13)\end{aligned}$$

with $v_0 \triangleq \min\{\lambda_2 - k_1, \delta_1, \delta_2 \ldots, \delta_N\}$, where in the second inequality, we have applied Lemma 1 and the definition of $\dot{\sigma}_i$. By applying the Comparison Principle [22, Lemma 3.4], we can obtain that V is bounded. According to the definition of V, it can be obtained that the dynamic variables $\sigma_i(t)$ are uniformly bounded, the vector χ is bounded and thus the closed-loop MAS can reach bounded consensus. □

C. Inter-Event Analysis

The following theorem shows that the proposed DET sampling strategy in (3)–(5), ensures a strictly positive minimum for the inter-event intervals T_k^i.

Theorem 2. *Consider the MAS (1) under Assumptions 1 and 2, the protocol (2) and the DET sampling mechanisms in (3)–(5). Then,* $\inf_{k\in\mathbb{N}}\{T_k^i\} > 0$ *for all* $i \in \mathcal{V}$.

Proof. To describe the triggering process in (3)–(5), we define two comparison functions as

$$\bar{\phi}_i = \frac{4l_{ii}\tilde{e}_i^2}{\rho_i\sum_{j\in\mathcal{N}_i}a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 + o_{\mathrm{f}}^i + \omega_i\sigma_i},\ \phi_i = \frac{\tilde{e}_i^2}{\sigma_i}.$$

Note that at the event instant t_k^i, both $\bar{\phi}_i$ and ϕ_i equal zero. Then the inter-event time $T_k^i = t_{k+1}^i - t_k^i$ resulting from the DET mechanisms in (3)–(5) is the time that it takes for the function $\bar{\phi}_i$ to evolve from 0 to 1 for the first time after t_k^i. Moreover, there obviously holds

$$\bar{\phi}_i \leq \frac{4l_{ii}\tilde{e}_i^2}{\omega_i\sigma_i} = \frac{4l_{ii}}{\omega_i}\phi_i.$$

Thus, the inter-event time T_k^i is lower bounded by the time it takes for the function ϕ_i to evolve from 0 to $\frac{\omega_i}{4l_{ii}}$ for the first time after t_k^i. To estimate this lower bound, taking the time derivative of ϕ_i and substituting $\dot{e}_i$ in (6) and $\dot{\sigma}_i$ in (4) into it, we can obtain

$$\begin{aligned}\dot{\phi}_i &= \frac{2\tilde{e}_i\dot{\tilde{e}}_i}{\sigma_i} - \frac{\tilde{e}_i^2}{\sigma_i^2}\dot{\sigma}_i \\ &= \frac{2\tilde{e}_i\sum_{j\in\mathcal{N}_i}a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)}{\sigma_i} - \frac{2\tilde{e}_iv_i}{\sigma_i} - \frac{2\tilde{e}_i\dot{w}_i}{\sigma_i} + \delta_i\frac{\tilde{e}_i^2}{\sigma_i} \\ &\quad + \frac{\tilde{e}_i^2}{\sigma_i^2}\left[4l_{ii}\tilde{e}_i^2 - \rho_i\sum_{j\in\mathcal{N}_i}a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - o_\sigma^i\right]. \quad (14)\end{aligned}$$

From Young's Inequality, it holds that

$$\frac{2\tilde{e}_i(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)}{\sigma_i} - \rho_i\frac{\tilde{e}_i^2}{\sigma_i^2}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 \leq \frac{1}{\rho_i}, \quad (15)$$

and

$$\begin{aligned}&\frac{2\tilde{e}_iv_i}{\sigma_i} + \frac{2\tilde{e}_i\dot{w}_i}{\sigma_i} - \frac{\tilde{e}_i^2}{\sigma_i^2}o_\sigma^i \\ &\leq \frac{2|\tilde{e}_i|(\bar{v}_0^i + \bar{w}_2^i)}{\sigma_i} - \frac{\tilde{e}_i^2}{\sigma_i^2}o_\sigma^i \leq \frac{(\bar{v}_0^i + \bar{w}_2^i)^2}{o_\sigma^i} \quad (16)\end{aligned}$$

Substituting (15) and (16) into (14) and using the definition of ϕ_i, we can obtain that

$$\dot{\phi}_i \leq \frac{l_{ii}}{\rho_i} + \frac{(\bar{v}_0^i + \bar{w}_2^i)^2}{o_\sigma^i} + \delta_i\phi_i + 4l_{ii}\phi_i^2.$$

Then by the Comparison Principle [22, Lemma 3.4], there holds $\phi_i(t) \leq \psi_i(t)$ for all $t \geq 0$, where $\psi_i(t)$ is the solution to the differential equation

$$\begin{aligned}&\dot{\psi}_i = \frac{l_{ii}}{\rho_i} + \frac{(\bar{v}_0^i + \bar{w}_2^i)^2}{o_\sigma^i} + \delta_i\psi_i + 4l_{ii}\psi_i^2, \\ &\psi(t_k^{i+}) = \phi(t_k^{i+}) = 0.\end{aligned}$$

Note that the time needed by ψ_i to go from 0 to $\frac{\omega}{4l_{ii}}$ is

$$T_i = \int_0^{\frac{\omega_i}{4l_{ii}}} \frac{1}{\frac{l_{ii}}{\rho_i} + \frac{(\bar{v}_0^i + \bar{w}_2^i)^2}{o_\sigma^i} + \delta_is + 4l_{ii}s^2}\mathrm{d}s > 0, \quad (17)$$

which is a strictly positive constant. According to the previous discussions, T_i is also a lower bound of the time that $\bar{\phi}_i$ need to evolve from 0 to 1. Hence, we can obtain that T_k^i of the sequence $\{t_k^i\}$ are lower bounded as $T_k^i > T_i$, $\forall k \in \mathbb{N}$, which implies that $\inf_{k\in\mathbb{N}}\{T_k^i\} \geq T_i > 0$. □

Remark 1. If $\bar{v}_0^i$ and $\bar{w}_2^i$ in Assumption 2 are known, T_i in (17) can be explicitly obtained by

$$T_i = \begin{cases}\frac{2}{\sqrt{\Delta_i}}\left(\arctan\frac{2cs_0+b}{\sqrt{\Delta_i}} - \arctan\frac{b}{\sqrt{\Delta_i}}\right), & \Delta_i > 0, \\ \frac{2}{b} - \frac{2}{2cs_0+b}, & \Delta_i = 0, \\ \frac{1}{\sqrt{-\Delta_i}}\left(\ln\frac{b+\sqrt{-\Delta_i}}{b-\sqrt{-\Delta_i}} - \ln\frac{2cs_0+b+\sqrt{-\Delta_i}}{2cs_0+b-\sqrt{-\Delta_i}}\right), & \Delta_i < 0,\end{cases}$$

where $s_0 = \frac{\omega_i}{4l_{ii}}$ and $\Delta_i \triangleq 4ac - b^2$ with

$$a = \frac{l_{ii}}{\rho_i} + \frac{(\bar{v}_0^i + \bar{w}_2^i)^2}{o_\sigma^i},\ b = \delta_i,\ c = 4l_{ii}.$$

This provides a method for explicitly computing T_i when $\bar{v}_0^i$ and $\bar{w}_2^i$, $i \in \mathcal{V}$ are known. This is also adopted in the simulation part of this paper in Section IV for illustrating the effectiveness of the derived lower bound T_i.

Remark 2. Consider the case when the external disturbances v_i and w_i, $i \in \mathcal{V}$ do not exist. According to the proofs of Theorems 1 and 2, it is not difficult to verify that the relevant parameters in (3)–(5) can be set as $\rho_i = 1$ and $o_{\mathrm{f}}^i = o_\sigma^i = 0$ while the consensus of MASs is achieved and positive MIETs are still ensured. Then the DET mechanisms in (3)–(5) can be rewritten in form of

$$f_i(t) = 4l_{ii}\tilde{e}_i^2 - \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - \omega_i\sigma_i(t),$$
$$\dot{\sigma}_i(t) = -\delta_i\sigma_i + \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - 4l_{ii}\tilde{e}_i^2,$$

which is similar to the DET mechanisms proposed in [7] but has some substantial differences. The first main difference is that the parameter ρ_i in [7] is required to belong to $(0, 1)$ while our design in $f_i(t)$ corresponds to $\rho_i = 1$. Since a larger ρ_i implies that the triggering function can be delayed to reach zero, our design is better at enlarging triggering intervals. Moreover, it should be pointed out that Theorem 2 theoretically establishes the existence of strictly positive MIETs for the designed DET conditions, while only Zeno-freeness has been proved in [7], which is another main difference of our design. On one hand, it is seen that the choice of ρ_i can be relaxed from being strictly smaller than 1 in the presence of disturbances to being equal to 1 in the absence of disturbances. On the other hand, changing ρ_i from 1 to $(0, 1)$ is a modification for robustifying the inter-event properties of DET mechanisms against external disturbances.

Remark 3. As discussed in Remark 2, in the absence of disturbances, the parameters o_{f}^i and o_σ^i can be set to zero while still ensuring the event-separation properties of the DET mechanisms. However, when subject to external disturbances, a positive o_σ^i becomes essential to guarantee strictly positive MIETs as shown in the proof of Theorem 2. Therefore, it is essential to emphasize that o_{f}^i and o_σ^i serve as critical parameters (as well as the modification of the coefficient ρ_i) that enhance the event-separation properties of DET mechanisms against external disturbances.

Remark 4. Recently, a space-regularization technique was reported in [5] to ensure positive inter-event times when the measurement noises w_i are bound but are not required to be differentiable, that is, $\dot{w}_i(t)$ are not limited to be bound. Following the technique proposed in [5], define $e_i \triangleq \hat{x}_i - x_i$ with $\hat{x}_i(t) = x_i(t_k^i)$, $\hat{w}_i(t) = w_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$, and it can be verified that

$$\begin{aligned} f_i(t) &= 4l_{ii}\tilde{e}_i^2 - \rho_i \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2 - o_{\mathrm{f}}^i - \omega_i\sigma_i(t) \\ &< 4l_{ii}(\hat{x}_i - x_i + \hat{w}_i - w_i)^2 - o_{\mathrm{f}}^i \\ &\le 8l_{ii}(\hat{x}_i - x_i)^2 + 8l_{ii}(\hat{w}_i - w_i)^2 - o_{\mathrm{f}}^i \\ &\le 8l_{ii}e_i^2 + 32l_{ii}(\bar{w}_1^i)^2 - o_{\mathrm{f}}^i. \end{aligned} \tag{18}$$

Since bounded consensus of MASs can be ensured (note that the proof of Theorem 1 does not require the boundness of $\dot{w}_i$), it can be concluded that the derivative of e_i^2 is bounded by some positive constants $\bar{e}$. Choose o_{f}^i such that $o_{\mathrm{f}}^i > 32l_{ii}(\bar{w}_1^i)^2$. Then according to (18), a guaranteed lower bound of MIETs, τ_0, is given by

$$\tau_0 = \frac{o_{\mathrm{f}}^i - 32l_{ii}(\bar{w}_1^i)^2}{8l_{ii}\bar{e}} > 0. \tag{19}$$

Therefore, for disturbances with known bounds $\bar{w}_1^i$, positive MIETs can be ensured by simply selecting $o_{\mathrm{f}}^i > 32l_{ii}(\bar{w}_1^i)^2$. However, note that the proposed DET mechanisms (3)–(5) are not a special case of those in [5]. Firstly, the considered DET mechanisms in [5] are different from the ones in (3)–(5). Secondly, Assumption 2 is not a special case of the one in [5] because the above modification requires a *known* bound $\bar{w}_1^i$ while Assumption 2 does *not*. Thirdly, as is seen from the lower bound in (19) and its derivation, it is the regularizer o_{f}^i, rather than a DET mechanism or the state-dependent term $\rho_i \sum_{j\in\mathcal{N}_i} a_{ij}(\hat{\tilde{x}}_i - \hat{\tilde{x}}_j)^2$, that essentially ensures the existence of a positive lower bound of inter-event times, which does not reflect the role of a DET mechanism, while the one in (17) clearly illustrates the joint role of o_σ^i and the DET mechanisms in (3)–(5), rather than o_{f}^i. Therefore, the proposed DET protocol provides new insights into the robustness of event-separation properties of distributed DET mechanisms.

IV. A Numerical Example

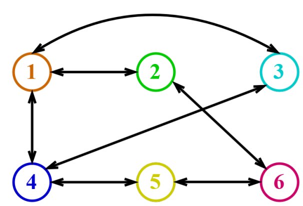

Fig. 1. The communication graph. All edge weights are one.

The MAS considered in the simulation consists of six agents, and the associated communication graph is shown in Fig. 1. Obviously, the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is undirected and connected, thereby satisfying Assumption 1. In the MAS (1), the external disturbances $v_i(t)$ and the derivative of $w_i(t)$, $i \in \mathcal{V}$ are randomly generated within the range of $[-0.1, 0.1]$. Additionally, $w_i(t)$ is artificially constrained to the interval $[-0.1, 0.1]$. Specifically, $w_i(t)$ is set to 0.1 if $w_i(t) \ge 0.1$ and is set to -0.1 if $w_i(t) \le -0.1$. For the DET mechanisms in (3)–(5), let $\rho_i = 0.5$, $o_{\mathrm{f}}^i = 0.25$, $\omega_i = 2.5 + 0.5i$, $\delta_i = 0.1l_{ii}$, $o_\sigma^i = 0.5$, $\sigma_i(0) = 1$, $i \in \mathcal{V}$. Moreover, the initial states of the agents are randomly selected in $[-1, 1]$.

The simulation results of the agents' actual states x_i and the measured states $\tilde{x}_i$ are shown in Figs. 2-A and B, respectively. It is clear that the closed-loop MAS reaches bounded consensus. Shown in Fig. 2-C are the dynamic variables $\sigma_i(t)$, $i \in \mathcal{V}$. It can be observed that $\sigma_i(t)$, $i \in \mathcal{V}$, are all uniformly bounded. To further illustrate the sampling sequences under the DET mechanisms in (3)–(5), Fig. 3 plots the inter-event times for each agent, where the results on average inter-event times $\mathrm{mean}_k\{T_k^i\}$, MIETs $\min_k\{T_k^i\}$ and guaranteed lower bounds T_i in (17) are displayed. It is shown that the resulting inter-event times T_k^i are uniformly lowered bounded and moreover the obtained T_i provides an effective lower bound of T_k^i.

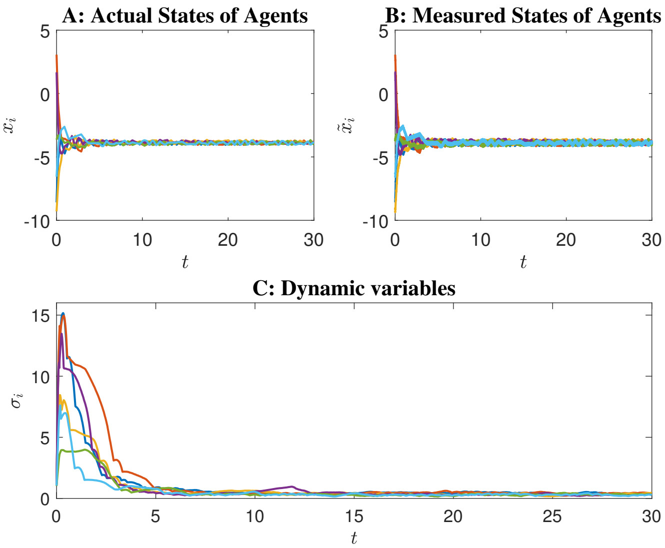

Fig. 2. The actual/measured states of agents and dynamic variables.

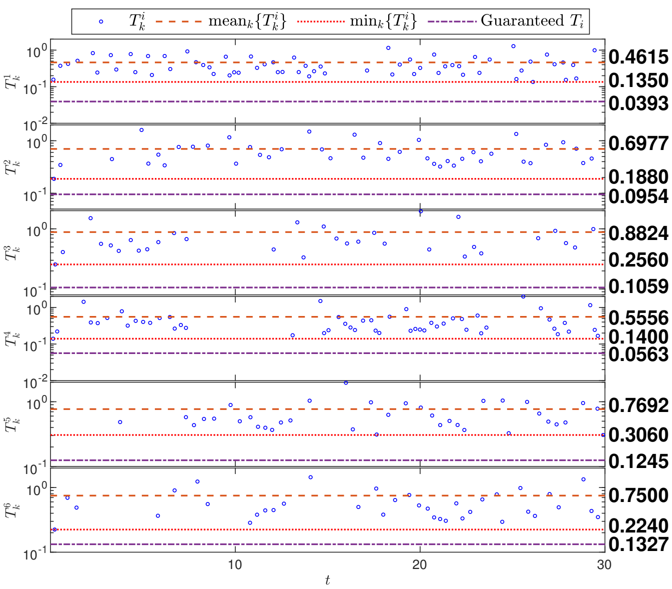

Fig. 3. Inter-event times T_k^i, average inter-event times $\mathrm{mean}_k\{T_k^i\}$, MIETs $\min_k\{T_k^i\}$ and guaranteed T_i. The horizontal and vertical coordinates of each circle "$\circ$" are the event instants and event intervals, respectively.

V. Conclusion

This paper has studied the consensus problem for MASs subject to external disturbances. Distributed protocols and DET sampling mechanisms are proposed and only intermittent communication between agents is required. The proposed DET sampling strategies enhance event-separation properties in two aspects. Firstly, unlike previous works where only Zeno behavior is excluded, the designed DET mechanisms have been shown to ensure positive MIETs. Secondly, even when external disturbances are taken into account, strictly positive inter-event times are still guaranteed by the proposed DET sampling schemes. Future work includes extending the proposed methods to directed graphs and linear MASs.

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